Problem 1 in Section 2.2. Consider the Differential Equation $\frac{dx}{dt} = x - 4$.

- What are the equilibrium solutions of this autonomous Differential Equation?
- Draw the phase diagram for this Differential Equation.
- Are the equilibrium solutions of this Differential Equation stable or unstable?
- Sketch the solution of the Initial Value problem

  $$\frac{dx}{dt} = x - 4 \quad x(0) = x_0,$$

  for a few choices of x_0.
- Solve the Differential Equation. (Be sure that your answer is in the form x is equal to some function of t.)

We see that $x = 4$ is the only equilibrium solution of the Differential Equation. We see that $x - 4$ is positive if $4 < x$ and $x - 4$ is negative if $x < 4$. (We drew the Phase diagram on the last page of this solution.) We conclude that the equilibrium solution is unstable. That is if $x(0)$ is more than 4, then $x(t)$ increases away from 4 as t gets large. Also if $x(0)$ is less than 4, then $x(t)$ decreases away from 4 as t gets large. We drew a picture of $x(t)$ for three choices of $x(0)$ on the next page. We chose one $x(0)$ to be larger than 4, one to be equal to 4, and one to be smaller than four.

Now we solve the Differential Equation. Separate the variables and integrate:

$$\int \frac{dx}{x-4} = \int dt$$

$$\ln |x - 4| = t + C$$

Exponentiate:

$$|x - 4| = e^C e^t$$

$$x - 4 = \pm e^C e^t$$

Let K be the constant $\pm e^C$.

$$x - 4 = Ke^t$$

Plug in $t = 0$:

$$x_0 - 4 = K$$

The solution of the Initial Value Problem

$$\frac{dx}{dt} = x - 4 \quad x(0) = x_0$$

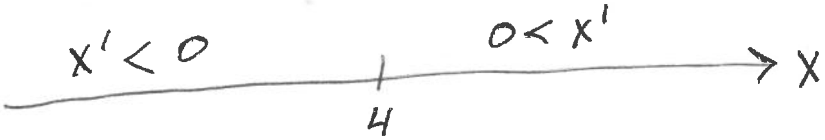
is

$$\boxed{x = 4 + (x_0 - 4)e^t}.$$

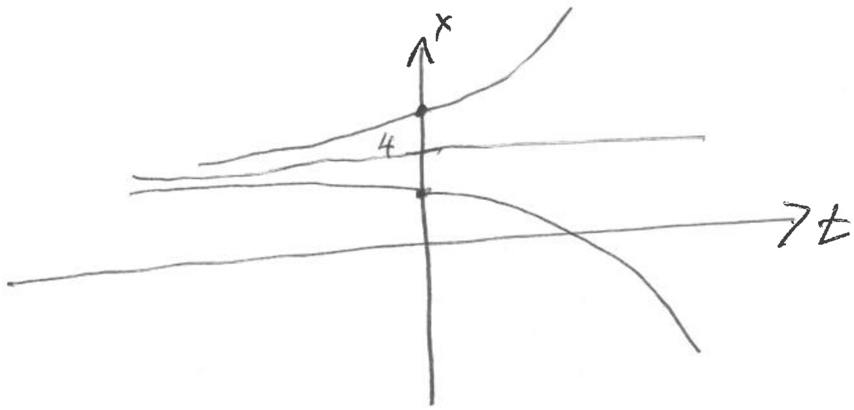
Check: We see that $x(0) = 4 + (x_0 - 4) = x_0$. We also see that $x' = (x_0 - 4)e^t$. On the other hand, $x - 4$ is also equal to $4 + (x_0 - 4)e^t - 4 = (x_0 - 4)e^t$. Thus, our proposed solution does satisfy the Differential Equation.

The pictures are on the next page.

Problem 1 in 2.2



Phase Diagram



A few solutions of $x' = x - 4$ $x(0) = x_0$