Problem 9 in Section 2.1. The time rate of change of a rabbit population P is proportional to the square rrot pf P. At time $t = 0$ (months) the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be one year later?

Solution. We are told that $\frac{dP}{dt} = k\sqrt{P}$, $P(0) = 100$, and $\frac{dP}{dt}(0) = 20$. We must find $P(12)$.

We can evaluate k at the beginning. Indeed, $\frac{dP}{dt}(0) = k\sqrt{P(0)}$; thus, $20 = k\sqrt{100}$. It follows that $20 = 10k$ and $k = \frac{20}{10} = 2$.

We separate the variables and integrate:

$$\int \frac{dP}{\sqrt{P}} = \int k\,dt$$

$$2\sqrt{P} = kt + C$$

. We can evaluate C by plugging $t = 0$ into both sides of the equation:

$$2\sqrt{100} = C$$

and

$$20 = C.$$

At this point we have

$$2\sqrt{P} = 2t + 20.$$

Divide both sides by 2; then square each side:

$$P = (t + 10)^2.$$

It follows that $P(12) = (22)^2$.

After one year there will be $(22)^2$ rabbits.