Problem 15 in Section 2.1 Consider a population $P(t)$ which satisfies the logistic equation $\frac{dP}{dt} = aP - bP^2$, where a and b are constants, $B = aP$ is the birth rate, and $D = bP^2$ is the death rate. Write M in terms of $B(0)$, $D(0)$, and $P(0)$.

Comment The point of the problem is that we know the solution of

$$\tfrac{dP}{dt} = kP(M - P).$$

In particular, we know that in the long term the population will approach M.

Someone interested in the population can probably calculate $B(0)$, $D(0)$, and $P(0)$. If we can express M in terms of $B(0)$, $D(0)$, and $P(0)$, then we can make a plausible prediction of what the population will do without doing anymore calculating.

Solution. Look at the two equations:

$$\frac{dP}{dt} = kMP - kP^2$$
$$\frac{dP}{dt} = aP - bP^2$$

The coefficient of P^2 is $-k$ in the top equation and is $-b$ in the bottom equation; so, $k = b$. The coefficient of P is kM in the top equation and is a in the bottom equation. Thus $a = kM$; so $a = bM$ and $\frac{a}{b} = M$.

Plug 0 into $B = aP$ and $D = aP^2$ to learn that $B(0) = aP(0)$ and $D(0) = bP(0)^2$. We conclude that

$$M = \frac{a}{b} = \frac{\frac{B(0)}{P(0)}}{\frac{D(0)}{P(0)^2}} = \frac{B(0)P(0)}{D(0)}.$$

Our answer is

$$\boxed{M = \frac{B(0)P(0)}{D(0)}.}$$