Problem 43 in Section 1.4. A pitcher of buttermilk initially at $25°$C is to be cooled by setting it on the front porch, where the temperature is $0°$C. Suppose that the temperature of the buttermilk has dropped to $15°$C after 20 min. When will it be at $5°$C?

Solution. Newton's Law of Cooling states that the rate at which an object cools is proportional to difference between the temperature of the object and the temperature of the surrounding medium.

Let $T(t)$ be the temperature of the buttermilk at time t. Measure T in degrees C and t in minutes. Take $t = 0$ to be the time when the buttermilk was put on the porch. Newton's Law of cooling says that $\frac{dT}{dt} = k(0 - T)$. We are told $T(0) = 25$ and $T(20) = 15$.

Separate the variables and integrate to see that

$$\int \frac{dT}{T} = -k \int dt$$

$$\ln T = -kt + C$$

(The temperature is always at least 0, so $|T| = T$.) Exponentiate:

$$T = e^C e^{-kt}.$$

Plug in $t = 0$ to see that $25 = T(0) = e^C$. Thus,

$$T(t) = 25e^{-kt}.$$

Plug in $t = 20$ to see that

$$15 = T(20) = 25e^{-k20}$$

$$\frac{15}{25} = e^{-k20}$$

$$\ln(\tfrac{3}{5}) = -k20$$

$$\frac{\ln \frac{3}{5}}{-20} = k$$

The temperature of the buttermilk will be 5 degrees C when

$$5 = 25e^{-kt}$$

$$\tfrac{1}{5} = e^{-kt}$$

$$\ln(\tfrac{1}{5}) = -kt$$

$$t = \frac{\ln(\frac{1}{5})}{-k} = \boxed{\frac{20 \ln \frac{1}{5}}{\ln \frac{3}{5}} \text{ minutes}}$$

This number is about 63.