Problem 24 in Section 1.4. Solve the Initial Value Problem

$$(\tan x)\frac{dy}{dx} = y \quad \text{and} \quad y(\tfrac{\pi}{2}) = \tfrac{\pi}{2}.$$

Solution. Separate the variables:

$$(*) \quad \frac{dy}{y} = \frac{\cos x}{\sin x}\, dx.$$

Integrate both sides. On the right side, let $u = \sin x$. It follows that $du = \cos x\, dx$. So the right side becomes $\int \frac{du}{u} = \ln|u| + C$. At any rate when we integrate (*) we get

$$\ln|y| = \ln|\sin x| + C.$$

Exponentiate to obtain

$$|y| = e^C |\sin x|$$
$$y = \pm e^C \sin x.$$

Now we evaluate the constant $\pm e^C$ by using $y(\frac{\pi}{2}) = \frac{\pi}{2}$:

$$\tfrac{\pi}{2} = y(\tfrac{\pi}{2}) = (\pm e^C)\sin\tfrac{\pi}{2} = (\pm e^C).$$

So, the constant $(\pm e^C)$ is equal to $\frac{\pi}{2}$ and the solution of the Initial Value Problem is

$$\boxed{y = \tfrac{\pi}{2}\sin x.}$$

Check. Plug

$$y = \tfrac{\pi}{2}\sin x$$
$$\frac{dy}{dx} = \tfrac{\pi}{2}\cos x$$

into the left side of the Differential Equation and obtain

$$(\tan x)\frac{dy}{dx}$$
$$= (\tan x)\frac{\pi}{2}\cos x$$
$$= \frac{\pi}{2}\sin x = y\checkmark$$

and $y(\frac{\pi}{2}) = \frac{\pi}{2}\sin(\frac{\pi}{2}) = \frac{\pi}{2}.\checkmark$.