Problem 15 in Section 1.3. What does the existence and uniqueness theorem tell you about the Initial Value Problem:

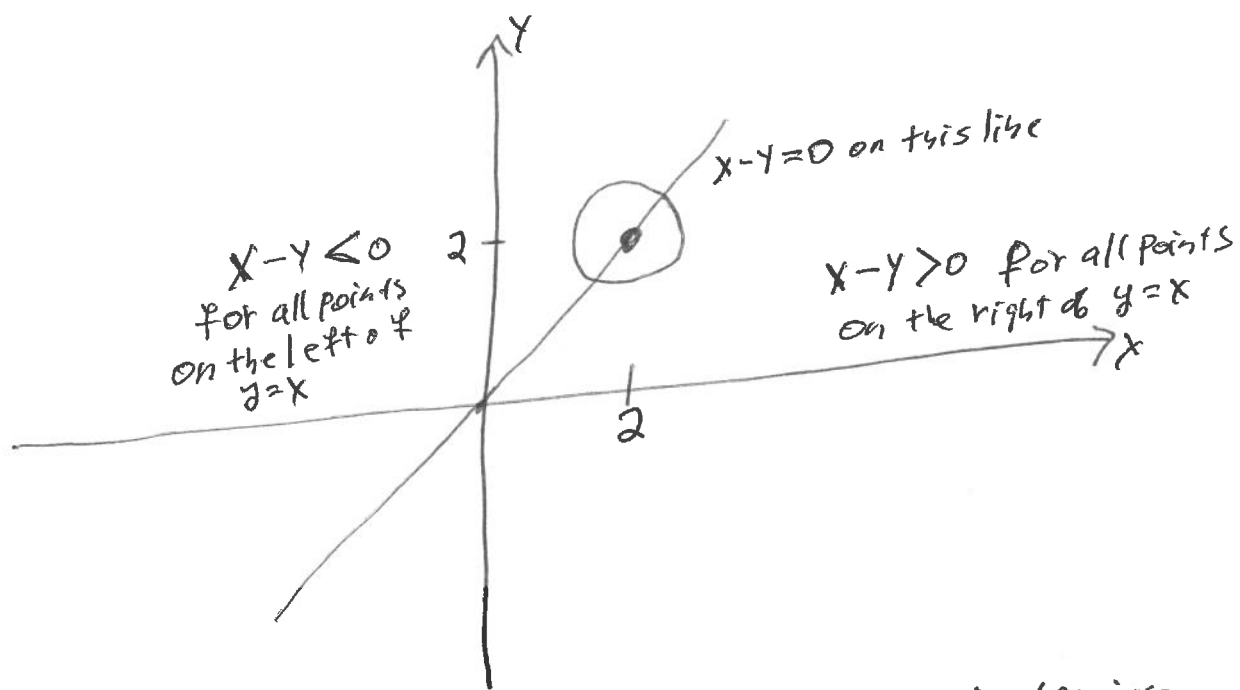$$\frac{dy}{dx} = \sqrt{x - y} \quad \text{and} \quad y(2) = 2,$$

if anything?

Solution.

View the Differential Equation as $\frac{dy}{dx} = f(x, y)$, where $f(x, y) = \sqrt{x - y}$. Notice that f is continuous when $x > y$. Unfortunately, ever every circle which contains $(2, 2)$ in its interior, also contains points with x-coordinate smaller than y-coordinate. (Draw a circle with $(2, 2)$ in its interior. Draw the line $y = x$. Notice that some points inside your circle are on the left side of the line $y = x$.) There is a picture on the next page.

> The Existence and Uniqueness Theorem does not tell us anything about the Initial Value Problem.

Y

$X - Y = 0$ on this line

$X - Y < 0$ for all points on the left of $y = x$

2

$X - Y > 0$ for all points on the right of $y = x$

X

2

Every circle which has $(2,2)$ in its interior also contains some points with $X - Y < 0$.