Problem 13 in Section 1.3. What does the existence and uniqueness theorem tell you about the Initial Value Problem:

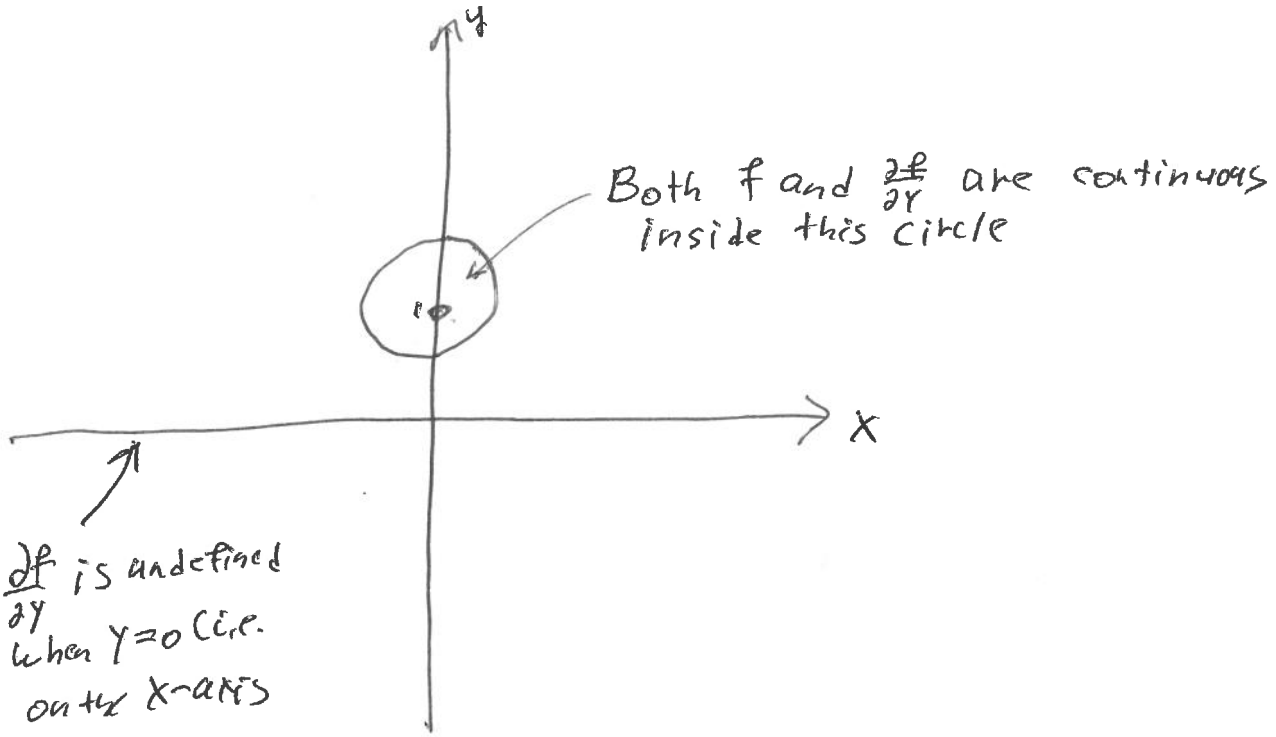
$$\frac{dy}{dx} = y^{1/3} \quad \text{and} \quad y(0) = 1,$$

if anything?

Solution. View the Differential Equation as $\frac{dy}{dx} = f(x, y)$, where $f(x, y) = y^{1/3}$. We see that f is continuous everywhere, and $\frac{\partial f}{\partial y} = -(1/3)y^{-2/3}$ is continuous everywhere, except at $y = 0$. We can draw a circle which has $(0, 1)$ in its interior which does not include any points on the x-axis. (For example, the circle with center $(0, 1)$ and radius $1/2$.) There is a picture on the next page.

The Existence and Uniqueness Theorem Guarantees that the Initial Value Problem has a unique solution.

Picture for Problem 13 in Section 1.3



The point • is (0, 1)