

Problem 11 in Section 1.3. What does the existence and uniqueness theorem tell you about the Initial Value Problem:

$$\frac{dy}{dx} = 2x^2y^2 \quad \text{and} \quad y(1) = -1,$$

if anything?

Solution. View the Differential Equation as $\frac{dy}{dx} = f(x, y)$, where $f(x, y) = 2x^2y^2$. Observe that $f(x, y)$ is continuous everywhere. Calculate $\frac{\partial f}{\partial y} = 4x^2y$. This function also is continuous everywhere.

The Existence and Uniqueness Theorem Guarantees that the Initial Value Problem has a unique solution.