Problem 6 in Section 1.2. Solve the Initial Value Problem

$$\frac{dy}{dx} = x\sqrt{x^2+9} \quad \text{and} \quad y(-4) = 0.$$

Solution. Separate the variables and integrate:

$$\int 1\, dy = \int x\sqrt{x^2+9}\, dx.$$

Let $u = x^2 + 9$. Then $du = 2x\, dx$. We obtain

$$y = \tfrac{1}{2}\int u^{\frac{1}{2}}\, du = \tfrac{1}{2}\tfrac{2}{3}u^{3/2} + C = \tfrac{1}{3}(x^2+9)^{3/2} + C$$

We use the initial condition $0 = y(-4)$ to find C:

$$0 = y(-4) = \tfrac{1}{3}((-4)^2+9)^{3/2} + C = \tfrac{125}{3} + C$$

So, $C = -\frac{125}{3}$ and

$$\boxed{y = \tfrac{1}{3}(x^2+9)^{3/2} - \tfrac{125}{3}.}$$

Check. We compute

$$y = \tfrac{1}{3}(x^2+9)^{3/2} - \tfrac{125}{3}$$
$$y' = \tfrac{1}{2}(x^2+9)^{1/2}2x\checkmark$$

and $y(-4) = \frac{1}{3}(25)^{3/2} - \frac{125}{3} = 0\checkmark$