Problem 33 in Section 1.2. On the planet Gzyx, a ball dropped from a height of 20 feet hits the ground in 2 seconds. If a ball is dropped from the top of a 200 foot tall building, how long will it take to hit the ground? With what speed will it hit the ground? (The fact from Physics is that there is a positive constant g_1 so that the acceleration of each ball is $-g_1$.)

Solution. Measure distance in feet and time in seconds. Let $x_1(t)$ be the height of the first ball at time t and $x_2(t)$ be the height of the second ball at time t. We know that

$$x_1(0) = 20, \quad x_1'(0) = 0, \quad x_1''(t) = -g_1, \quad x_1(2) = 0.$$

Integrate twice and evaluate the constants:

$$x_1'(t) = -g_1 t + C_1$$

$$0 = x_1'(0) = C_1$$

$$x_1'(t) = -g_1 t$$

$$x_1(t) = -g_1 \tfrac{t^2}{2} + C_2$$

$$20 = x_1(0) = C_2$$

$$x_1(t) = -g_1 \tfrac{t^2}{2} + 20$$

The fact that $x_1(2) = 0$ tells us that

$$0 = -g_1(2) + 20.$$

We conclude that $g_1 = 10$ ft/sec^2.

Now we think about the second ball. We have

$$x_2(0) = 200, \quad x_2'(0) = 0, \quad \text{and} \quad x_2'' = -g_1 = -10.$$

We must find the value of t for which $x_2(t) = 0$. Integrate and evaluate the constants to obtain:

$$x_2'(t) = -10t$$

$$x_2(t) = -10\tfrac{t^2}{2} + 200.$$

We solve

$$0 = x(t) = -5t^2 + 200$$

to see that the ball hits the ground after $\boxed{t = \sqrt{40} \text{ seconds}}$ and the speed of the ball when it hits the ground is $x_2'(\sqrt{40}) = \boxed{-10\sqrt{40} \text{ ft/sec}}$. (The minus sign indicates that the ball is going downward.)