

Problem 32 in Section 1.2. Suppose that a car skids 15 m if it is moving at 50 km/h when the brakes are applied. Assuming that the car has the same constant deceleration, how far will it skid if it is moving at 100 km/hr when the brakes are applied?

Suggestion. I suggest that you ignore the meters, kilometers, and hours from the given problem and calculate the stopping distance as a function of the initial speed v_0 and the constant deceleration $-k$. (Notice that k is positive.) Then see how the stopping distance changes when v_0 is replaced by $2v_0$.

Solution. Let x be the position of the car at time t . Let $t = 0$ be the moment the brakes are applied. Take $x(0) = 0$. Let $-k$ be the constant deceleration of this car. (Notice that k is a positive number.) Let v_0 be the speed of the car when the brakes are applied. Our plan is to find a formula for the stopping distance in terms of k and v_0 . Let t_{stop} be the time when the car stops. Of course, $x(t_{\text{stop}})$ is how far the car skidded.

The Initial Value Problem is $x'' = -k$, $x'(0) = v_0$, and $x(0) = 0$. The solution of the IVP is $x(t) = -kt^2/2 + v_0t$ and $x'(t) = -tk + v_0$. The car stops when $x'(t_{\text{stop}}) = 0$; so $0 = -kt_{\text{stop}} + v_0$. So the car stops at $t_{\text{stop}} = \frac{v_0}{k}$. The position of the car when it stops is $x(t_{\text{stop}}) = -k(\frac{v_0}{k})^2/2 + v_0(\frac{v_0}{k}) = \frac{v_0^2}{2k}$. **If the initial speed is doubled then the stopping distance is multiplied by a factor of 4.** In particular if the initial speed is raised from 50 k/h to 100 k/h, then the stopping distance increases from 15 m to $60m$.