Problem 16 in Section 1.2. The position of an object at time t is $x(t)$. The acceleration of the object is $x''(t) = \frac{1}{\sqrt{t+4}}$, the initial velocity is $x'(0) = -1$ and the initial position is $x(0) = 1$. Find the formula for $x(t)$.

Solution. Integrate to learn that

$$x' = 2(t+4)^{1/2} + C_1$$

$$x = \tfrac{4}{3}(t+4)^{3/2} + C_1 t + C_2$$

Plug in the initial conditions to see that

$$-1 = x'(0) = 2(2) + C_1$$

(so $C_1 = -5$) and

$$1 = x(0) = (\tfrac{4}{3})8 + C_2$$

(so $\frac{-29}{3} = C_2$).

$$\boxed{x = \tfrac{4}{3}(t+4)^{3/2} - 5t - \tfrac{29}{3}}$$

Check. We compute

$$x' = 2(t+4)^{1/2} - 5,$$
$$x = (t+4)^{-1/2}\checkmark,$$
$$x(0) = \tfrac{4}{3}(8) - \tfrac{29}{3} = 1\checkmark,$$

and

$$x'(0) = 2(2) - 5 = -1\checkmark.$$