**Problem 10 in Section 1.2.** Solve the Initial Value Problem

$$\frac{dy}{dx} = xe^{-x} \quad \text{and} \quad y(0) = 1.$$

**Solution.** We use Integration by Parts:

$$\int u\,dv = uv - \int v\,du.$$

Take $u = x$ and $dv = e^{-x}$. Compute $du = dx$ and $v = -e^{-x}$. Separate the variables and integrate

$$\int 1\,dy = \int xe^{-x}\,dx$$

$$y = -xe^{-x} - \int -e^{-x}\,dx$$

$$y = -xe^{-x} - e^{-x} + C$$

Plug in $1 = y(0)$ to see that

$$1 = y(0) = -1 + C.$$

Conclude that

$$\boxed{y = -xe^{-x} - e^{-x} + 2}.$$

**Check.** We compute $y' = -x(-e^{-x}) - e^{-x} + e^{-x} = xe^{-x}$. ✓and $y(0) = 0 - 1 + 2 = 1$. ✓