Problem 45 in Section 1.1. Suppose a population of rodents satisfies the Differential Equation $\frac{dP}{dt} = kP^2$. Initially, there are $P(0) = 2$ rodents, and their number is increasing at the rate of $\frac{dP}{dt} = 1$ rodent per months when there are $P = 10$ rodents. How long will it take for this population to grow to one hundred rodents? To a thousand? What is happening here?

Solution. We can evaluate the constant k. We are told that $\frac{dP}{dt} = kP^2$ and when $P = 10$, then $\frac{dP}{dt} = 1$. It follows that $1 = k(100)$ and $k = \frac{1}{100}$.

Separate the variables:

$$P^{-2}dP = kdt$$

and integrate

$$\int P^{-2}\,dP = \int k\,dt$$

$$\frac{-1}{P} = kt + C$$

This is a good time to evaluate C. When $t = 0$, $P = 2$; so

$$\frac{-1}{2} = C$$

$$\frac{-1}{kt + \frac{-1}{2}} = P$$

$$\frac{2}{1 - 2kt} = P.$$

We already saw that $k = \frac{1}{100}$; thus,

$$P = \frac{2}{1 - 2(\frac{1}{100})t}$$

$$P = \frac{100}{50 - t}$$

There will be 100 rodents when $100 = \frac{100}{50-t}$;

$$50 - t = 1$$

$$\boxed{t = 49 \text{ months}}$$

There will be 1000 rodents when $50 - t = \frac{1}{10}$; so $\boxed{t = 49.9 \text{ months}}$

$$\boxed{\text{The rat population is going to infinity as time goes to 50 months.}}$$