Problem 30 in Section 1.1. Consider the functions $y = g(x)$ which have the property that the graph of g is perpendicular to every curve of the form $y = x^2 + k$ (k is a constant) wherever they meet. Write a Differential Equation of the form $\frac{dy}{dx} =$ some function of x and y, which has g as one of its solutions.

Solution. Let (x_0, y_0) be a point on the graph one of the functions $y = g(x)$ which has the desired property. On the one hand, the slope of the line tangent to $y = g(x)$ at the point (x_0, y_0) is $g'(x_0)$. On the other hand (x_0, y_0) is also on the curve $y = x^2 + k$, where k is the constant $y_0 - x_0^2$. The slope of the parabola $y = x^2 + k$ at (x_0, y_0) is $2x_0$. The problem states that the line tangent to the parabola at (x_0, y_0) is perpendicular to the line tangent to $y = g(x)$ at (x_0, y_0). The product of the slopes of these tangent lines is -1. Thus

$$(2x_0)g'(x_0) = -1.$$

In other words, $y = g(x)$ is a function with

$$2x_0 y'(x_0) = -1$$

for all x_0. We conclude that $y = g(x)$ is a solution of the differential equation

$$\boxed{2xy' = -1}.$$