Problem 26 in Section 1.1. Verify that $y(x) = (x + C)\cos x$ is a solution of the Differential Equation $y' + y\tan x = \cos x$ for any constant C. Find the constant C for which $y(\pi) = 0$.

Solution. We plug

$$y = (x + C)\cos x \quad \text{and} \quad y' = (x + C)(-\sin x) + \cos x$$

into $y' + y\tan x$ and obtain

$$(x + C)(-\sin x) + \cos x + (x + C)\cos x \tan x$$

$$= -\sin x(x + C) + \cos x + (x + c)\sin x$$

$$= \cos x \checkmark$$

If $y(\pi) = 0$, then $0 = (\pi + C)\cos(\pi)$; so $0 = (\pi + C)(-1)$ and $C = -\pi$.

We conclude that $y = (x - \pi)\cos x$ is a solution of the initial value problem $y' + y\tan x = \cos x$ and $y(\pi) = 0$.