Problem 16 in Section 1.1. Find all constants r so that $y = e^{rx}$ is a solution of $y'' + 3y' - 4y = 0$.

Solution. We calculate $y' = re^{rx}$ and $y'' = r^2e^{rx}$. When we put y, y', and y'' into the differential equation we obtain

$$3r^2e^{rx} + 3re^{rx} - 4e^{rx} = 0.$$

Factor to obtain

$$e^{rx}(3r^2 + 3r - 4) = 0.$$

If a product is equal to zero, then one of the factors must be zero. The factor e^{rx} is never zero; so, $3r^2 + 3r - 4 = 0$. We use the quadratic formula: If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In our problem, $a = 3$, $b = 3$, and $c = -4$; Thus,

$$r = \frac{-3 \pm \sqrt{9 - 4(3)(-4)}}{2(3)}.$$

In other words,

$$\boxed{r = \frac{-3 \pm \sqrt{57}}{6}.}$$