**Problem 11 in Section 1.1.** Check that $y_1 = \frac{1}{x^2}$ and $y_2 = \frac{\ln x}{x^2}$ are solutions of the Differential Equation $x^2y'' + 5xy' + 4y = 0$, where $y' = \frac{dy}{dx}$.

**Solution.** We plug

$$\begin{aligned} y_1 &= x^{-2} \\ y_1' &= -2x^{-3} \\ y_1'' &= 6x^{-4} \end{aligned}$$

into $x^2y'' + 5xy' + 4y$ and obtain

$$x^2(6x^{-4}) + 5x(-2x^{-3}) + 4x^{-2} = x^{-2}(6 - 10 + 4) = 0.\checkmark$$

We plug

$$\begin{aligned} y_2 &= \frac{\ln x}{x^2} \\ y_2' &= \ln x(-2)x^{-3} + x^{-3} \\ y_2'' &= \ln x(6x^{-4}) - 2x^{-4} - 3x^{-4} = 6x^{-4}\ln x - 5x^{-4} \end{aligned}$$

into $x^2y'' + 5xy' + 4y$ and obtain

$$x^2(6x^{-4}\ln x - 5x^{-4}) + 5x((-2)x^{-3}\ln x + x^{-3}) + 4x^{-2}\ln x$$

$$= x^{-2}\ln x(6 - 10 + 4) + x^{-2}(-5 + 5) = 0\checkmark$$

We see that $y_1$ and $y_2$ both are solutions of the Differential Equation.