5. Compute $\displaystyle\int_1^3\int_{-y}^{2y} xe^{y^3}\,dx\,dy$.

We see that

$$\int_1^3\int_{-y}^{2y} xe^{y^3}\,dx\,dy = \int_1^3 \frac{x^2}{2}e^{y^3}\Big|_{-y}^{2y} dy = \frac{1}{2}\int_1^3 3y^2e^{y^3}\,dy = \frac{1}{2}\ e^{y^3}\Big|_1^3 dy = \boxed{\frac{1}{2}(e^{27}-e).}$$

6. Compute $\displaystyle\int_0^4\int_{\sqrt{x}}^{2} \sin(y^3)dy\,dx$ **.**

We can not express $\int \sin(y^3)\,dy$ in terms of elementary functions. The present integral uses vertical lines to fill up the region. We switch and use horizontal lines. See the picture on a separate page. The integral is equal to

$$\int_0^2\int_0^{y^2} \sin(y^3)dx\,dy = \int_0^2 x\sin(y^3)\Big|_0^{y^2} dy = \int_0^2 y^2\sin(y^3)dy = -\frac{1}{3}\cos(y^3)\Big|_0^2$$

$$= -\frac{1}{3}(\cos 8 - 1) = \boxed{\frac{1}{3}(1-\cos 8).}$$