

Quiz for March 7, 2008

Find the equation of the plane tangent to $z = 4x^3y^2 + 2y$ at the point $P = (1, -2, 12)$.

Answer: Gradients are perpendicular to level sets. Write our surface as $0 = 4x^3y^2 + 2y - z$. Our surface is the level set $F = 0$ for $F = 4x^3y^2 + 2y - z$. We want the plane through $P = (1, -2, 12)$ which is perpendicular to

$$\nabla F(1, -2, 12) = (12x^2y^2\vec{i} + (8x^3y + 2)\vec{j} - \vec{k})(1, -2, 12) = 48\vec{i} - 14\vec{j} - \vec{k}.$$

The tangent plane is:

$$\boxed{48(x - 1) - 14(y + 2) - (z - 12) = 0.}$$