

Quiz for February 22, 2008

Find the curvature of the curve parametrized by $\vec{r}(t) = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$ at $t = 1$.

Answer: We know

$$\kappa(t) = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} = \frac{\left\| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 3t^2 - 2 & 2t & 0 \\ 6t & 2 & 0 \end{array} \right\|}{\|(3t^2 - 2)^2 + 4t^2\|^{3/2}} = \frac{\|[2(3t^2 - 2) - 12t^2]\vec{k}\|}{\|(3t^2 - 2)^2 + 4t^2\|^{3/2}};$$

so,

$$\kappa(1) = \frac{\|[2(3 - 2) - 12]\vec{k}\|}{\|(3 - 2)^2 + 4\|^{3/2}} = \boxed{\frac{10}{5^{3/2}}}.$$