Quiz for February 1, 2008

Consider the two lines

$$L_1: \; x+1=4t,\; y-3=t,\; z-1=0$$

$$L_2: \; x+13=12t,\; y-1=6t,\; z-2=3t.$$

(a) Find the point of intersection of the two lines.

(b) Find the plane determined by the two intersecting lines. **CHECK your answer.**

First we work on (a). Suppose t_1 puts L_1 at the point (x,y,z) and t_2 puts L_2 at the same point (x,y,z). We first find t_1 and t_2. These values of t must satisfy all three of the equations:

$$\begin{cases} 4t_1 - 1 = 12t_2 - 13 \\ t_1 + 3 = 6t_2 + 1 \\ 1 = 3t_2 + 2 \end{cases}$$

The bottom equation tells us that $t_2 = -1/3$. The middle equation tells us that $t_1 = -4$. These values work the first equation since: $4(-4) - 1 = 12(-1/3) - 13$. The point of intersection is $\boxed{(-17,-1,1)}$. (This point satisfies equations for both L_1 and L_2.)

Now we work on (b). We want the plane through $(-17,-1,1)$ that is parallel to $<4,1,0>$ and $<12,6,3>$. We want the plane through $(-17,-1,1)$ that is normal to

$$<4,1,0> \times <12,6,3> = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 4 & 1 & 0 \\ 12 & 6 & 3 \end{vmatrix}$$

$$= \bar{i}\begin{vmatrix} 1 & 0 \\ 6 & 3 \end{vmatrix} - \bar{j}\begin{vmatrix} 4 & 0 \\ 12 & 3 \end{vmatrix} + \bar{k}\begin{vmatrix} 4 & 1 \\ 12 & 6 \end{vmatrix} = 3\bar{i} - 12\bar{j} + 12\bar{k}.$$

The plane is

$$3(x+17) - 12(y+1) + 12(z-1) = 0$$

or

$$\boxed{x - 4y + 4z + 9 = 0.}$$

We see that line L_1 is on the proposed plane because:

$$(4t-1) - 4(t+3) + 4(1) + 9 = 0$$

for all t. We see that line L_2 is on the proposed plane because:

$$(12t-13) - 4(6t+1) + 4(3t+2) + 9 = 0$$

for all t.