Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will return your quiz when I next see you.

The quiz is worth 5 points. The solution will be posted on my website later today.

Quiz 8, November 20, 2024

Change the integral into polar coordinates, then integrate:

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\,dy\,dx$$

(You must draw the region.)

Answer: Look at the picture on the next page. The region for this integral is defined by: For each fixed θ with $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, r goes from $r=0$ to $r=\sqrt{2}$. In polar coordinates x becomes $r\cos\theta$ and y becomes $r\sin\theta$, and $dy\,dx$ becomes $r\,dr\,d\theta$. The original integral is equal to

$$\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{2}} (r\cos\theta + 2r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\pi/4}^{\pi/2}\int_0^{\sqrt{2}} (r^2\cos\theta + 2r^2\sin\theta)\,dr\,d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left(\tfrac{r^3}{3}\cos\theta + \tfrac{2r^3}{3}\sin\theta\right)\Big|_0^{\sqrt{2}}\,d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left(\tfrac{2^{3/2}}{3}\cos\theta + \tfrac{2(2^{3/2})}{3}\sin\theta\right)d\theta$$

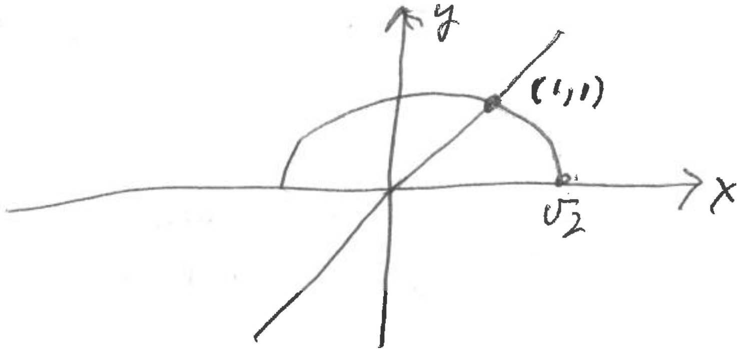
$$= \left(\tfrac{2^{3/2}}{3}\sin\theta - \tfrac{2(2^{3/2})}{3}\cos\theta\right)\Big|_{\pi/4}^{\pi/2}$$

$$= \tfrac{2^{3/2}}{3} - 0 - \tfrac{2^{3/2}}{3}\tfrac{\sqrt{2}}{2} + \tfrac{2(2^{3/2})}{3}\tfrac{\sqrt{2}}{2}$$

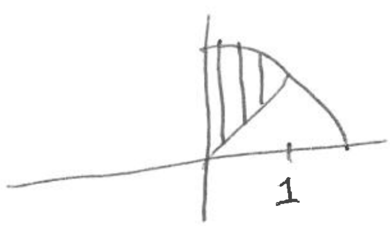
$$= \boxed{\tfrac{2^{3/2}+2}{3}}$$

The picture for $\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\, dy\, dx$

We draw $y=x$ and $y=\sqrt{2-x^2}$. Of course $y=\sqrt{2-x^2}$ is the part of the circle $x^2+y^2=2$ where y is positive



For each fixed x with $0 \le x \le 1$, y goes from $y=x$ to y on the circle



Of course this region is also described by For each fixed θ with $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, r goes from $r=0$ to $r=\sqrt{2}$.