Homework for Chapter 14

- 14.1, number 1c: Let $f(x,y) = x^2 + xy^3$. Find $f(2,3)$.
- 14.1, number 5: Find and sketch the domain of $f(x,y) = \sqrt{y-x-2}$.
- 14.1, number 15: Sketch the level sets $f(x,y) = c$ for $f(x,y) = xy$ and c equals -9, -4, -1, 0, 1, 4, 9.
- 14.1, number 18: Let $f(x,y) = \sqrt{y-x}$.
  (a) Find the domain of f.
  (b) Find the image of f.
  (c) Describe f's level sets.
  (d) Find the boundary of f's domain.
  (e) Is the boundary open region, or a closed region, or neither?
  (f) Is the domain bounded or unbounded?
- 14.1, number 39: Let $f(x,y) = x^2 + y^2$.
  (a) Graph the surface $z = f(x,y)$.
  (b) Draw a few level sets of f.
- 14.1, number 47: Let $f(x,y) = \sqrt{x^2 + y^2 + 4}$.
  (a) Graph the surface $z = f(x,y)$.
  (b) Draw a few level sets of f.
- 14.1, number 49: Sketch the level set for $f(x,y) = 16 - x^2 - y^2$ which passes through $(2\sqrt{2}, \sqrt{2})$. What is the equation of this level set?
- 14.1, number 53: Sketch a typical level set for the function

$$f(x,y,z) = x^2 + y^2 + z^2.$$

- 14.1, number 61: Find the equation of the level set of

$$f(x,y,z) = \sqrt{x-y} - \ln z$$

  which passes through $(3,-1,1)$.
- 14.2, number 1: Find $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$.
- 14.2, number 13: Find $\lim_{(x,y)\to(0,0)} \frac{x^2-2xy+y^2}{x-y}$.
- 14.2, number 33: Where is $g(x,y) = \sin\frac{1}{xy}$ continuous?

- 14.2, number 41: Show that

$$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}}$$

  does not exist.

- 14.2, number 43: Show that $\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ does not exist.

- 14.3, number 1: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = 2x^2 - 3y - 4$.

- 14.3, number 7: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = \sqrt{x^2+y^2}$.

- 14.3, number 19: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = x^y$.

- 14.3, number 27: Find f_x, f_y, and f_z, for $f(x,y,z) = \arcsin(xyz)$.

- 14.3, number 35: Find the partial derivative of $f(t,\alpha) = \cos(2\pi t - \alpha)$ with respect to each variable.

- 14.3, number 41: Find all second partial derivatives of

$$f(x,y) = x + y + xy.$$

- 14.3, number 55: Verify that $w_{xy} = w_{yx}$ for $w = \ln(2x+3y)$.

- 14.4, number 1: Consider $w = x^2 + y^2$, $x = \cos t$, and $y = \sin t$.

  (a) Use the chain rule to calculate $\frac{dw}{dt}$.

  (b) First write w as a function of t directly, then compute $\frac{dw}{dt}$ using first semester calculus techniques.

  (c) Evaluate $\frac{dw}{dt}(\pi)$.

- 14.4, number 7: Consider $z = 4e^x \ln y$, $x = \ln(u\cos v)$, and $y = u \sin v$.

  (a) Use the chain rule to calculate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$. Express your answers in terms of u and v. (In other words, there should be no x's and y's in your answer.)

  (b) First write z as a function of u and v directly, then compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

  (c) Evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u,v) = (2, \frac{\pi}{4})$.

- 14.4, number 37: Find $\frac{\partial w}{\partial v}$ at $(u,v) = (0,0)$ if $w = x^2 + \frac{y}{x}$, $x = u - 2v + 1$, and $y = 2u + v - 2$.

- 14.5, number 6: Find the gradient of $f(x,y) = \arctan\left(\frac{\sqrt{x}}{y}\right)$ at the point $P = (4,-2)$. Draw the level set of f that passes through P. Draw the gradient; put its tail on P.

- 14.5, number 9: Let $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2} + \ln(xyz)$ and $P = (-1, 2, -2)$. Find $(\overrightarrow{\nabla} f)|_P$.

- 14.5, number 11: Let $f(x, y) = 2xy - 3y^2$, $P = (5, 5)$, and $\overrightarrow{v} = 4\overrightarrow{i} + 3\overrightarrow{j}$. Find $D_{\overrightarrow{v}} f|_P$. (That is, find the directional derivative of f in the direction of $\overrightarrow{v}$ at the point P.)

- 14.5, number 19: Let $f(x, y) = x^2 + xy + y^2$ and $P = (-1, 1)$.

  (a) Which direction gives the largest directional derivative of f at P? What is the directional derivative of f at P in that direction?

  (b) Which direction gives the smallest directional derivative of f at P? What is the directional derivative of f at P in that direction?

- 14.5, number 25: Let $f(x, y) = x^2 + y^2$. Consider the level set $f(x, y) = 4$ and the point $P = (\sqrt{2}, \sqrt{2})$ on that level set. Sketch

  (a) the level set $f(x, y) = 4$,

  (b) $\overrightarrow{\nabla} f|_P$ (Put the tail of the gradient on P.), and

  (c) the line tangent to the level set $f(x, y) = 4$ at P.

  What is the equation of the line tangent to the level set $f(x, y) = 4$ at P?

- 14.6, number 1: Find the equation of the plane tangent to $x^2 + y^2 + z^2 = 3$ at the point $P = (1, 1, 1)$. Find parametric equations for the line normal to $x^2 + y^2 + z^2 = 3$ at the point $P = (1, 1, 1)$.

- 14.6, number 11: Find the equation of the plane tangent to $z = \ln(x^2 + y^2)$ at the point $P = (1, 0, 0)$.

- 14.6, number 15: Let C be the curve which is the intersection of the two surfaces $x + y^2 + 2z = 4$ and $x = 1$. Give parametric equations for the line that is tangent to C at $(1, 1, 1)$.

- 14.7, number 1: Find all local maxima, local minima, and saddle points of $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

- 14.7, number 7: Find all local maxima, local minima, and saddle points of $f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$.

- 14.7, number 13: Find all local maxima, local minima, and saddle points of $f(x, y) = x^3 - y^3 - 2xy + 6$.

- 14.7, number 21: Find all local maxima, local minima, and saddle points of $f(x, y) = \frac{1}{x^2+y^2-1}$.

- 14.7, number 31: Find the absolute maxima and absolute minima of $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.

- 14.7, number 33: Find the absolute maxima and absolute minima of $f(x,y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.

- 14.7, F20-e3-4: Find the local maximum points, local minimum points, and saddle points of $f(x,y) = x^2y + 4xy - 2y^2$.

- 14.7, F20-e3-5: Find the absolute extreme points of the function

  $$f(x,y) = x + y - xy,$$

  which is defined on the closed triangle with vertices at $(0,0)$, $(0,2)$, and $(4,0)$.

- 14.7, S19-e3-4: Find the absolute maximum and absolute minimum of

  $$f(x,y) = 2 + 2x + 4y - x^2 - y^2$$

  on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

- 14.7, F18-e3-1: Find all local minima, local maxima and saddle points for the function $f(x,y) = x^2 + 4y^2 - 6x + 8y - 15$.

- 14.7, F18-e3-2: Find the absolute maximum and the absolute minimum values of

  $$f(x,y) = 3xy - 6x - 3y + 7$$

  on the triangular region with vertices $(0,0)$, $(3,0)$, and $(0,5)$.

- 14.7, F17-e3-11:40-section-4: Find the absolute maximum and minimum values of

  $$f(x,y) = 2 + 2x + 4y - x^2 - y^2$$

  on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

- 14.8, number 1: Find the points on the ellipse $x^2 + 2y^2 = 1$ where the function $f(x,y) = xy$ has its extreme values.

- 14.8, number 5: Find the points on the curve $xy^2 = 54$ nearest the origin.

- 14.8, number 9: Find the dimensions of the closed right circular cylindrical can of smallest surface area whose volume is 16π cubic centimeters.

- 14.8, number 12: Find the dimensions of the rectangle of largest perimeter that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{x^2}{b^2} = 1$ with sides parallel to the coordinate axes? What is the largest perimeter?

- 14.8, number 23: Find the maximum and minimum values of

  $$f(x, y, z) = x - 2y + 5z$$

  on the sphere $x^2 + y^2 + z^2 = 30$.