## Homework for 13.1 – 13.3

- 13.1, number 5: The position vector of an object at time $t$ is $\vec{\boldsymbol{r}}(t) = (t+1)\vec{\boldsymbol{i}} + (t^2-1)\vec{\boldsymbol{j}}$. Eliminate $t$ and find an equation involving only $x$ and $y$ which gives the path of the object. Find the object's velocity and acceleration vectors at time $t = 1$.

- 13.1, number 7: The position vector of an object at time $t$ is $\vec{\boldsymbol{r}}(t) = e^t\vec{\boldsymbol{i}} + \frac{2}{9}e^{2t}\vec{\boldsymbol{j}}$. Eliminate $t$ and find an equation involving only $x$ and $y$ which gives the path of the object. Find the object's velocity and acceleration vectors at time $t = \ln 3$.

- 13.1, number 9: The position vector of an object at time $t$ is $\vec{\boldsymbol{r}}(t) = (\sin t)\vec{\boldsymbol{i}} + (\cos t)\vec{\boldsymbol{j}}$. Find the velocity vector and the acceleration vector of the object at time $\pi/4$ and time $\pi/2$. Draw these vectors on the graph of the path of the object; put the tail of each vector on the position of the object at the given times. (Of course, the object is traveling on the circle $x^2 + y^2 = 1$.)

- 13.1, number 13: The position vector of an object at time $t$ is

  $$\vec{\boldsymbol{r}}(t) = (t+1)\vec{\boldsymbol{i}} + (t^2-1)\vec{\boldsymbol{j}} + 2t\vec{\boldsymbol{k}}.$$

  Find the object's velocity vector and acceleration vector. Find the object's speed and direction of motion at time $t = 1$. Write $\vec{\boldsymbol{v}}(1)$ as the object's speed at time $t = 1$ times a unit vector.

- 13.1, number 23: Find parametric equations for the line that is tangent the curve parameterized by $\vec{\boldsymbol{r}}(t) = (\sin t)\vec{\boldsymbol{i}} + (t^2 - \cos t)\vec{\boldsymbol{j}} + e^t\vec{\boldsymbol{k}}$ at $t = 0$.

- 13.1, number 37b: The position vector of a particle at time $t$ is given by $\vec{\boldsymbol{r}}(t) = \cos(2t)\vec{\boldsymbol{i}} + \sin(2t)\vec{\boldsymbol{j}}$, for $0 \le t$. (Of course the particle moves on the circle $x^2 + y^2 = 1$.)

  i) Does the particle have a constant speed? If so, what is it?
  ii) Is the particle's acceleration always orthogonal to its velocity vector?
  iii) Does the particle move clock-wise or counterclockwise around the circle?
  iv) Is the particle initially located at the point $(1, 0)$?

- 13.1, number 37e: The position vector of a particle at time $t$ is given by $\vec{\boldsymbol{r}}(t) = \cos(t^2)\vec{\boldsymbol{i}} + \sin(t^2)\vec{\boldsymbol{j}}$, for $0 \le t$. (Of course the particle moves on the circle $x^2 + y^2 = 1$.)

i) Does the particle have a constant speed? If so, what is it?

ii) Is the particle's acceleration always orthogonal to its velocity vector?

iii) Does the particle move clock-wise or counterclockwise around the circle?

iv) Is the particle initially located at the point $(1, 0)$?

- 13.1, number 39: A particle moves along the top of the parabola $y^2 = 2x$ from left to right at a constant speed of $5$ units per second. Find the velocity vector of the particle as it moves through the point $(2, 2)$.
- 13.2, number 1: Evaluate $\int_0^1 [t^3 \vec{\boldsymbol{i}} + 7\vec{\boldsymbol{j}} + (t+1)\vec{\boldsymbol{k}}]dt$.
- 13.2, number 9: Evaluate $\int_0^{\pi/2} [\cos t\, \vec{\boldsymbol{i}} - 2\sin 2t\, \vec{\boldsymbol{j}} + \sin^2 t\, \vec{\boldsymbol{k}}]dt$.
- 13.2, number 11: Find $\vec{\boldsymbol{r}}(t)$ if $\frac{d\vec{\boldsymbol{r}}}{dt} = -t\,\vec{\boldsymbol{i}} - t\,\vec{\boldsymbol{j}} - t\,\vec{\boldsymbol{k}}$ and $\vec{\boldsymbol{r}}(0) = \vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}$.
- 13.2, number 17: Find $\vec{\boldsymbol{r}}(t)$ if $\frac{d^2\vec{\boldsymbol{r}}}{dt^2} = -32\vec{\boldsymbol{k}}$, $\vec{\boldsymbol{r}}(0) = 100\vec{\boldsymbol{k}}$, and $\left.\frac{d\vec{\boldsymbol{r}}}{dt}\right|_{t=0} = 8\vec{\boldsymbol{i}} + 8\vec{\boldsymbol{j}}$.
- 13.2, number 21: At time $t = 0$, a particle is located at the point $(1, 2, 3)$. It travels in a straight line to the point $(4, 1, 4)$, has speed 2 at $(1, 2, 3)$ and has constant acceleration $3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$. Find an equation for the position vector $\vec{\boldsymbol{r}}(t)$ of the particle at time $t$.
- 13.2, number 23: A projectile is fired at a speed of 840 m/sec at an angle of $60$ degrees. How long will it take to get 21 km down range?
- 13.2, number 24:

  a. Show that doubling a projectile's initial speed at a given launch angel multiplies its range by a factor of $4$.

  b. By about what percentage should you increase the initial speed to double the height and the range?
- 13.2, number 32: The picture (at the end of the problem set) shows an experiment with two marbles. Marble $A$ was launched toward marble $B$ with launch angle $\alpha$ and initial speed $v_0$. At the same instant, marble $B$ was released to fall from rest at $R\tan\alpha$ units directly above a spot $R$ units downrange from $A$. The marbles were found to collide regardless of the value of $v_0$. Was this mere coincidence, or must this happen? Give reasons for your answer.

- 13.3, number 1: Find the length of the curve

  $$\vec{\boldsymbol{r}}(t) = (2\cos t)\vec{\boldsymbol{i}} + (2\sin t)\vec{\boldsymbol{j}} + \sqrt{5}t\vec{\boldsymbol{k}},$$

  for $0 \leq t \leq \pi$.

- 13.3, number 9: Find the point on the curve

  $$\vec{\boldsymbol{r}}(t) = (5\sin t)\vec{\boldsymbol{i}} + (5\cos t)\vec{\boldsymbol{j}} + 12t\vec{\boldsymbol{k}}$$

  at a distance $26\pi$ units along the curve from the point $(0, 5, 0)$ in the direction corresponding to increasing $t$ values.

- 13.3, number 11: Find the length of the curve $\vec{\boldsymbol{r}}(t) = (4\cos t)\vec{\boldsymbol{i}} + (4\sin t)\vec{\boldsymbol{j}} + 3t\vec{\boldsymbol{k}}$ for $0 \leq t \leq \pi/2$.

- 13.3, number 15: Find the length of the curve

  $$\vec{\boldsymbol{r}}(t) = (\sqrt{2}t)\vec{\boldsymbol{i}} + (\sqrt{2}t)\vec{\boldsymbol{j}} + (1 - t^2)\vec{\boldsymbol{k}},$$

  from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

Picture for Section 13.2 Number 32