12.3 This problem is problem 3, from Exam 2, from Fall 2024. **Express $\vec{v} = 3\vec{i} + \vec{j}$ as the sum of a vector parallel to $\vec{b} = 4\vec{i} + 8\vec{j}$ plus a vector perpendicular to $\vec{b}$. Check your answer. Make sure it is correct.**

Answer: There is a picture on the next page. We compute

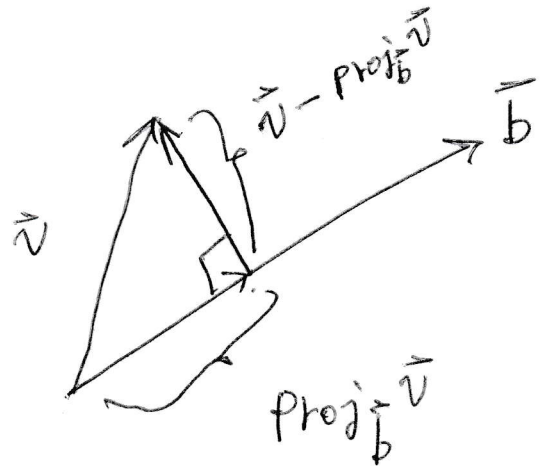
$$\begin{aligned}\text{proj}_{\vec{b}}\,\vec{v} &= \frac{\vec{v}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b} \\ &= \frac{12+8}{16+64}(4\vec{i}+8\vec{j}) = \frac{1}{4}(4\vec{i}+8\vec{j}) = \vec{i}+2\vec{j}.\end{aligned}$$

Observe that

$$\vec{v} - (\vec{i}+2\vec{j}) = 2\vec{i} - \vec{j}.$$

The vector $\vec{v} = 3\vec{i} + \vec{j}$ is equal to $\vec{i} + 2\vec{j}$ plus $2\vec{i} - \vec{j}$ with $\vec{i} + 2\vec{j}$ parallel to $\vec{b} = 4\vec{i} + 8\vec{j}$
and $2\vec{i} - \vec{j}$ perpendicular to $\vec{b} = 4\vec{i} + 8\vec{j}$.

Check. We verify the three assertions:
$(\vec{i} + 2\vec{j}) + (2\vec{i} - \vec{j}) = 3\vec{i} + \vec{j}$ ✓
$(\vec{i} + 2\vec{j}) = \frac{1}{4}(4\vec{i} + 8\vec{j})$ ✓
$(2\vec{i} - \vec{j}) \cdot (4\vec{i} + 8\vec{j}) = 8 - 8 = 0$ ✓


$\vec{v} - \text{Proj}_{\vec{b}}\vec{v}$
$\vec{b}$
$\vec{v}$
$\text{Proj}_{\vec{b}}\vec{v}$