12.3 This problem is problem 3, from Exam 1, from Fall 2024. **Express $\vec{v} = 2\vec{i} + 4\vec{j}$ as the sum of a vector parallel to $\vec{b} = \vec{i} + \vec{j}$ and a vector perpendicular to $\vec{b}$. Check your answer. Make sure it is correct.**

Answer: There is a picture on the next page. The vector $\vec{v}$ is equal to $\text{proj}_{\vec{b}}\,\vec{v}$ plus $\vec{v} - \text{proj}_{\vec{b}}\,\vec{v}$ with $\text{proj}_{\vec{b}}\,\vec{v}$ parallel to $\vec{b}$ and $\vec{v} - \text{proj}_{\vec{b}}\,\vec{v}$ perpendicular to $\vec{b}$ for

$$\text{proj}_{\vec{b}}\,\vec{v} = \frac{\vec{v}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b}.$$

We calculate

$$\text{proj}_{\vec{b}}\,\vec{v} = \frac{\vec{v}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b} = \frac{2+4}{1+1}\vec{b} = 3\vec{b} = 3\vec{i} + 3\vec{j}.$$

We calculate

$$\vec{v} - \text{proj}_{\vec{b}}\,\vec{v} = 2\vec{i} + 4\vec{j} - (3\vec{i} + 3\vec{j}) = -\vec{i} + \vec{j}.$$

We conclude that

$\vec{v} = (3\vec{i} + 3\vec{j}) + (-\vec{i} + \vec{j})$, with $3\vec{i} + 3\vec{j}$ parallel to $\vec{b}$ and $-\vec{i} + \vec{j}$ perpendicular to $\vec{b}$.

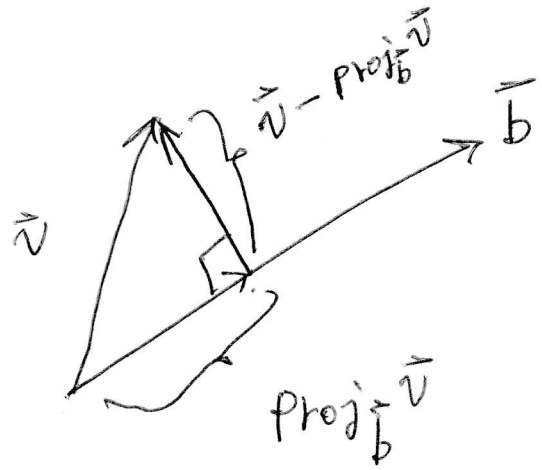
Check

Of course it is true that $(3\vec{i} + 3\vec{j}) + (-\vec{i} + \vec{j})$ is equal to $2\vec{i} + 4\vec{j}$, which is $\vec{v}$.✓

It is also true that $3\vec{i} + 3\vec{j}$ is parallel to $\vec{b} = \vec{i} + \vec{j}$.

We verify that $-\vec{i} + \vec{j}$ is perpendicular to $\vec{b}$:

$$(-\vec{i} + \vec{j})\cdot(\vec{i} + \vec{j}) = -1 + 1 = 0\checkmark.$$


$\vec{v} - \text{Proj}_{\vec{b}}\vec{v}$
$\vec{b}$
$\vec{v}$
$\text{Proj}_{\vec{b}}\vec{v}$