7. The temperature of a plate at the point (x, y) is $T(x,y) = 20 - 2x^2 - y^2$.

(a) Draw and label the level sets $T = -7$, $T = 0$, $T = 10$, and $T = 20$

(b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(3, 3)$. Draw the path of the particle.

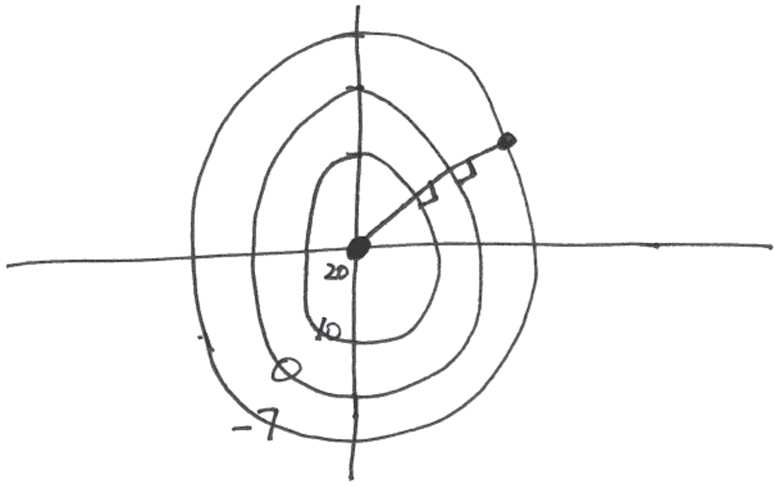
(c) Find the equation which gives the path of the particle of part (b).

$T=-7$ $\quad -7 = 20-2x^2-y^2$ $\quad 2x^2+y^2=27$ $\quad \frac{x^2}{13.5}+\frac{y^2}{27}=1$

$T=0$ $\quad 0=20-2x^2-y^2$ $\quad 2x^2+y^2=20$ $\quad \frac{x^2}{10}+\frac{y^2}{20}=1$

$T=10$ $\quad 10=20-2x^2-y^2$ $\quad 2x^2+y^2=10$ $\quad \frac{x^2}{5}+\frac{y^2}{10}=1$

$T=20$ $\quad 20=20-2x^2-y^2$ $\quad 2x^2+y^2=0$ $\quad (x,y)=(0,0)$



Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the position of the object at time t.

We know $\vec{r}'(t) = c(t)\vec{\nabla} T|_t$

$x'(t)\vec{i} + y'(t)\vec{j} = c(t)(-4x\vec{i} - 2y\vec{j})$

$$\frac{x'(t)}{-4x} = c(t) = \frac{y'(t)}{-2y}$$

$$\int \frac{x'(t)}{x}\,dt = \int \frac{2y'(t)}{y}\,dt$$

$$\ln|x| + C = 2\ln|y|$$

$$e^{\ln|x|+C} = e^{2\ln|y|}$$

$$e^C|x| = y^2$$

$kx = y^2$ where $k = \pm e^C$

$(3,3)$ is the initial condition

$k3 = 9$ $\qquad k=3$

$$\boxed{3x = y^2}$$