

Exam 4 Summer 2002 Math 241

PRINT Your Name: _____

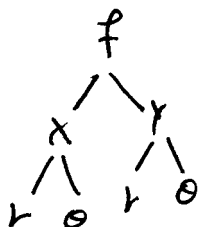
There are 8 problems on 4 pages. The exam is worth 50 points. SHOW your work.

CIRCLE your answer. **NO CALCULATORS!**

Neither your exam, nor your score, will not be available until class on Monday.

(7 points) Let $f(x, y) = 2x^2y^3 - x^3y^5$, $x = r \cos \theta$, and $y = r \sin \theta$. Find

$$\left. \frac{\partial f}{\partial \theta} \right|_{r=2, \theta=\pi/4}$$



$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \theta} = (4xy^3 - 3x^2y^5)(-r \sin \theta) + (6x^2y^2 - 5x^3y^4)(r \cos \theta)$$

$$x|_{r=2, \theta=\pi/4} = 2 \frac{\sqrt{2}}{2} = \sqrt{2}, \quad y|_{r=2, \theta=\pi/4} = 2 \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$\begin{aligned} \left. \frac{\partial f}{\partial \theta} \right|_{r=2, \theta=\pi/4} &= (4(\sqrt{2})(\sqrt{2})^3 - 3(2)(4\sqrt{2}))(-2 \frac{\sqrt{2}}{2}) + (6((2)(2)) - 5(2\sqrt{2})(4))(\frac{2\sqrt{2}}{2}) \\ &= (16 - 24\sqrt{2})(-\sqrt{2}) + (24 - 40\sqrt{2})\sqrt{2} \end{aligned}$$

$$48 - 80 + (-16 + 24)\sqrt{2}$$

$$= -32 + 8\sqrt{2}$$

2. (6 points) Let $f(x, y) = \frac{xy + y^3}{x^2 + y^2}$.

(a) Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along $y = x$.

(b) Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along $y = 2x$.

(c) What is $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$?

$$\textcircled{a} \lim_{\substack{(x, y) \rightarrow (0, 0) \\ \text{along } y=x}} f = \lim_{x \rightarrow 0} \frac{x^2 + x^3}{x^2 + x^2} = \lim_{x \rightarrow 0} \frac{x^2(1+x)}{2x^2} = \lim_{x \rightarrow 0} \frac{1+x}{2} = \frac{1}{2}$$

$$\textcircled{b} \lim_{\substack{(x, y) \rightarrow (0, 0) \\ \text{along } y=2x}} f = \lim_{x \rightarrow 0} \frac{2x^2 + 8x^3}{x^2 + 4x^2} = \lim_{x \rightarrow 0} \frac{2x^2(1+4x)}{5x^2} = \lim_{x \rightarrow 0} \frac{2(1+4x)}{5} = \frac{2}{5}$$

\textcircled{c} $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ does not exist because \textcircled{a} and \textcircled{b} are different.