

3. (6 points) Identify all local extreme points and all saddle points of $f(x, y) = xy^2 - 6x^2 - 3y^2$.

$$f'_x = y^2 - 12x$$

$$f'_y = 2xy - 6y$$

$f'_x = 0$ and $f'_y = 0$ when

$$y^2 - 12x = 0 \text{ and } y(2x - 6) = 0$$

so either $y = 0$ or $x = 3$

when $y = 0$, $y^2 - 12x = 0$ tells you

that $x = 0$

when $x = 3$, $y^2 - 12x = 0$ tells you

that $y^2 = 36$ so $y = 6$ or -6

there are 3 critical points

$(0, 0)$, $(3, 6)$, $(3, -6)$

$$f''_{xx} = -12 \quad f''_{yy} = 2x - 6$$

$$f''_{xy} = 2y$$

$$D = f''_{xx} f''_{yy} - (f''_{xy})^2$$

$$D = -12(2x - 6) - 4y^2$$

at $(0, 0)$

$$D|_{(0,0)} = -12(-6) - 0 > 0$$

$$f''_{xx}|_{(0,0)} = -12 < 0$$

$\therefore (0, 0, 0)$ is a local max

at $(3, 6)$

$$D|_{(3,6)} = 0 - 4(36) < 0$$

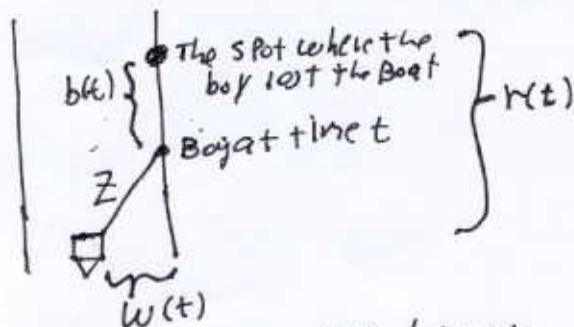
$\therefore (3, 6, 7)$ is a saddle point

at $(3, -6)$

$$D|_{(3,-6)} = 0 - 4(36) < 0$$

$\therefore (3, -6, 7)$ is a saddle point

4. (6 points) A boy's toy boat slips from his grasp at the edge of a straight river. The stream carries it along at 6 feet per second. A cross wind blows it toward the opposite bank at 5 feet per second. If the boy runs along the shore at 3 feet per second following his boat, how fast is the boat moving away from him 3 seconds after the boat slipped from his grasp.



$b(t)$ = the position of the boy at time t

$r(t)$ = the distance the river has pushed the boat down stream at time t

$w(t)$ = the distance the wind has pushed the boat toward the opposite shore at time t

$z(t)$ = The distance from the boy to the boat at time t

$$\text{We want } \frac{dz}{dt} \Big|_3$$

$$\text{We know } \frac{dr}{dt} = 6 \text{ ft/s}$$

$$\frac{dw}{dt} = 5 \text{ ft/s}$$

$$\frac{db}{dt} = 3 \text{ ft/s}$$

$$z^2 = (r - b)^2 + w^2$$

$$2z \frac{dz}{dt} = 2(r - b) \left(\frac{dr}{dt} - \frac{db}{dt} \right) + 2w \frac{dw}{dt}$$

$$\frac{dz}{dt} = \frac{(r - b) \left(\frac{dr}{dt} - \frac{db}{dt} \right) + w \frac{dw}{dt}}{z}$$

$$\frac{dz}{dt} \Big|_3 = \frac{(18 - 9)(6 - 3) + 15(5)}{\sqrt{9^2 + 15^2}} \quad 6/5$$

at $t = 3$:

