Math 241, Spring 1998, exam 3

PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Let $f(x,y,z) = xz\ln(x+y+z)$. Find $\overrightarrow{\nabla} f$.

2. Find the equation of the plane tangent to $z = x^2 + y^2$ at $(1,1,2)$.

3. Suppose that $z = f(x,y)$, and x and y are writen polar coordinates (that is, $x = r\cos\theta$ and $y = r\sin\theta$). Express $\dfrac{\partial z}{\partial \theta}$ in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. Find the directional derivative of $f(x,y,z) = xy + z^2$ at $(1,1,1)$ in the direction toward $(5,-3,3)$.

5. Let $f(x,y) = \dfrac{xy^2}{3x^2 + 2y^4}$.
   (a) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along every straight line of the form $y = mx$.
   (b) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the parabola $x = y^2$.
   (c) What is $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

6. Identify all local extreme points and all saddle points of $f(x,y) = x^3 + y^3 - 6xy$.

7. Graph $z = 9 - y^2 - x^2$ in $3-$ space.

8. Graph and label the level sets $f = 0$, $f = 10$, and $f = 20$ for $f(x,y) = 2x^2 - y^2 + 10$.

9. Graph $y^2 - \frac{x^2}{4} - \frac{z^2}{9} = 1$ in $3-$ space.

10. Find the absolute extreme points of $f(x,y) = x^2 - 6x + y^2 - 8y + 7$ on $\{(x,y) \mid x^2 + y^2 \le 1\}$.