Math 241, Spring 1998, exam 1

PRINT Your Name:____________________________

There are 10 problems on 4 pages. Each problem is worth 10 points. SHOW your work. $\boxed{\mathit{CIRCLE}}$ your answer. **NO CALCULATORS!** Check your answer whenever possible!

1. Graph $x^2 + y^2 = 1$ in $3-$ space.

2. Graph $x^2 + y^2 + z^2 = 1$ in $3-$ space.

3. Graph $x^2 = 1$ in $3-$ space.

4. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(1,2,1)$, $(2,0,2)$, and $(2,3,0)$.

5. Let $\overrightarrow{\boldsymbol{a}} = 2\overrightarrow{\boldsymbol{i}} + 3\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = 2\overrightarrow{\boldsymbol{i}} - \overrightarrow{\boldsymbol{j}} + 4\overrightarrow{\boldsymbol{k}}$. Find the angle between $\overrightarrow{\boldsymbol{a}}$ and $\overrightarrow{\boldsymbol{b}}$.

6. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equations of the line which contains $(1,2,3)$ and $(-4,2,0)$.

7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\overrightarrow{\boldsymbol{a}} = 2\overrightarrow{\boldsymbol{i}} + 3\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = 2\overrightarrow{\boldsymbol{i}} - \overrightarrow{\boldsymbol{j}} + 4\overrightarrow{\boldsymbol{k}}$. Find vectors $\overrightarrow{\boldsymbol{u}}$ and $\overrightarrow{\boldsymbol{v}}$ with $\overrightarrow{\boldsymbol{b}} = \overrightarrow{\boldsymbol{u}} + \overrightarrow{\boldsymbol{v}}$, $\overrightarrow{\boldsymbol{u}}$ parallel to $\overrightarrow{\boldsymbol{a}}$, and $\overrightarrow{\boldsymbol{v}}$ perpendicular to $\overrightarrow{\boldsymbol{a}}$.

8. Find a point which is the distance 2 from $x + 2y + 2z = 1$.

9. Find the point on $2x + y + 2z = 1$ which is closest to $(2,3,3)$.

10. The intersection of $x^2 + y^2 + z^2 \leq 9$ and $x^2 + y^2 + (z-5)^2 \leq 9$ is a solid in $3-$ space. Find the volume of this solid.