Math 241, Spring 2001, Final Exam

PRINT Your Name:____________________________

Get your course grade from **TIPS/VIP** late on Thursday or later.

There are 19 problems on 10 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 8 points. The exam is worth a total of 150 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!**

1. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(1,1,1)$, $(1,2,-2)$, and $(3,1,-3)$.

2. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equations of the line through $(1,3,4)$ and $(3,6,9)$.

3. Graph and name $y^2 - x^2 = 1$ in $2-$ space.

4. Graph and name $\frac{z^2}{16} - \frac{y^2}{9} + \frac{x^2}{25} = 1$ in $3-$ space.

5. What are the equations of the line tangent to the curve which is parameterized by $\vec{\boldsymbol{r}}(t) = (3t^3 + 2t)\,\vec{\boldsymbol{i}} + 6t^2\,\vec{\boldsymbol{j}} + 4t^3\,\vec{\boldsymbol{k}}$ at $t = 1$?

6. Find the equation of the plane tangent to the surface $z = x^2 + 3y^3$ at the point where $x = 3$ and $y = -1$.

7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{\boldsymbol{a}} = 2\,\vec{\boldsymbol{i}} + 4\,\vec{\boldsymbol{j}} + 6\,\vec{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 3\,\vec{\boldsymbol{i}} + 4\,\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$. Find vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$ with $\vec{\boldsymbol{b}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$, $\vec{\boldsymbol{u}}$ parallel to $\vec{\boldsymbol{a}}$, and $\vec{\boldsymbol{v}}$ perpendicular to $\vec{\boldsymbol{a}}$.

8. Find the point on $2x + 3y + 4z = 49$ which is closest to $(1,2,3)$.

9. Where do the following two lines intersect? CHECK YOUR ANSWER!

$$\frac{x-1}{1} = \frac{y-8}{2} = \frac{z-8}{3} \quad \text{and} \quad \frac{x+4}{-1} = \frac{y-10}{2} = \frac{z+4}{-2}$$

10. Find the length of the curve $\vec{\boldsymbol{r}}(t) = t^2\,\vec{\boldsymbol{i}} + t^3\,\vec{\boldsymbol{j}}$ for $1 \le t \le 2$.

11. Find the directional derivative of $f(x,y) = y^3 \ln x$ at the point $(1,2)$ in the direction of $\vec{\boldsymbol{u}} = \frac{1}{\sqrt{2}}(\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}})$.

12. Find all local maximum points, all local minimum points, and all saddle points of $f(x,y) = x^3 + y^3 - 6xy$.

13. The temperature of a plate at the point (x,y) is $T(x,y) = 20 - 3x^2 - 2y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(1,4)$. (The particle always moves in the direction of the greatest increase in temperature.)