Math 241, Spring 2001, Exam 2

PRINT Your Name:____________________________

There are 8 problems on 4 pages. Problems 1–4 are worth 13 points each. Each of the other problems is worth 12 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!**

1. Graph and describe the graph of $x^2 + y^2 + z^2 = 1$ in $3-$ space.

2. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equations of the line through $(1,4,7)$ and $(2,5,11)$.

3. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(1,1,2)$, $(3,1,1)$, and $(2,2,2)$.

4. Find the equations of the line which contains $(2,5,4)$ and is perpendicular to $3x + 7y + 4z = 8$.

5. What are the equations of the line tangent to the curve parameterized by $\overrightarrow{\boldsymbol{r}}(t) = 3t^3\overrightarrow{\boldsymbol{i}} + 2t^2\overrightarrow{\boldsymbol{j}} + 4t^5\overrightarrow{\boldsymbol{k}}$ at $t = -1$?

6. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\overrightarrow{\boldsymbol{a}} = -\overrightarrow{\boldsymbol{i}} + 2\overrightarrow{\boldsymbol{j}} + 3\overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = -3\overrightarrow{\boldsymbol{i}} + 5\overrightarrow{\boldsymbol{j}} + 5\overrightarrow{\boldsymbol{k}}$. Find vectors $\overrightarrow{\boldsymbol{u}}$ and $\overrightarrow{\boldsymbol{v}}$ with $\overrightarrow{\boldsymbol{b}} = \overrightarrow{\boldsymbol{u}} + \overrightarrow{\boldsymbol{v}}$, $\overrightarrow{\boldsymbol{u}}$ parallel to $\overrightarrow{\boldsymbol{a}}$, and $\overrightarrow{\boldsymbol{v}}$ perpendicular to $\overrightarrow{\boldsymbol{a}}$.

7. Find the equation of the plane which contains the point $(2,1,3)$ and the line

$$\begin{cases} x = 1 + t \\ y = -1 + 8t \\ z = 3 - 2t. \end{cases}$$

   CHECK YOUR ANSWER!

8. Find the length of the curve parameterized by $\overrightarrow{\boldsymbol{r}}(t) = \cos t\,\overrightarrow{\boldsymbol{i}} + \sin t\,\overrightarrow{\boldsymbol{j}} + t^{\frac{3}{2}}\overrightarrow{\boldsymbol{k}}$, for $0 \le t \le \frac{20}{3}$.