Math 241, Spring 2001, Exam 1

PRINT Your Name:______________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Graph and describe the graph of $xz = 0$ in $3-$ space.

2. Graph and describe the graph of $x^2 + z^2 = 0$ in $3-$ space.

3. Graph and describe the graph of $x^2 + z^2 = 1$ in $3-$ space.

4. Find the angle between $\overrightarrow{\boldsymbol{u}} = 3\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{v}} = 2\overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}} - \overrightarrow{\boldsymbol{k}}$.

5. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\overrightarrow{\boldsymbol{a}} = \overrightarrow{\boldsymbol{i}} + 2\overrightarrow{\boldsymbol{j}} + 3\overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = 4\overrightarrow{\boldsymbol{i}} + 6\overrightarrow{\boldsymbol{j}} + 4\overrightarrow{\boldsymbol{k}}$. Find vectors $\overrightarrow{\boldsymbol{u}}$ and $\overrightarrow{\boldsymbol{v}}$ with $\overrightarrow{\boldsymbol{b}} = \overrightarrow{\boldsymbol{u}} + \overrightarrow{\boldsymbol{v}}$, $\overrightarrow{\boldsymbol{u}}$ parallel to $\overrightarrow{\boldsymbol{a}}$, and $\overrightarrow{\boldsymbol{v}}$ perpendicular to $\overrightarrow{\boldsymbol{a}}$.

6. Find the equation of the plane which contains the point $(2,3,1)$ and is perpendicular to the vector $\overrightarrow{\boldsymbol{N}} = 2\overrightarrow{\boldsymbol{i}} - 3\overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}$.

7. Find the point on $(x-1)^2 + (y-3)^2 + (z+1)^2 = 14$ which is closest to $x + 2y + 3z = 30$.

8. Find the equation of the sphere whose center is $(2,4,5)$ and which is tangent to the xy-plane.

9. Find the work done by the force $\overrightarrow{\boldsymbol{F}} = 3\overrightarrow{\boldsymbol{i}} - 6\overrightarrow{\boldsymbol{j}} + 7\overrightarrow{\boldsymbol{k}}$ as it moves an object in a straight line from $(2,1,3)$ to $(9,4,6)$. Force is measured in pounds. Distance is measured in feet.

10. The vectors $\overrightarrow{\boldsymbol{v}}$ and $\overrightarrow{\boldsymbol{w}}$ live in the xy-plane. The vector $\overrightarrow{\boldsymbol{v}}$ has a magnitude of 60 and points in the direction $\frac{\pi}{12}$ radians. The vector $\overrightarrow{\boldsymbol{w}}$ has a magnitude of 80 and points in the direction $\frac{4\pi}{7}$ radians. How long is $\overrightarrow{\boldsymbol{v}} + \overrightarrow{\boldsymbol{w}}$? (Angles are measured counterclockwise starting at the positive x-axis. You may leave "cos" and/or "sin" in your answer.)