① $f(x,y) = x^2+y^2-x-y+1$

$f_x = 2x-1$

$f_y = 2y-1$

$f_x = f_y = 0$ when $(x,y) = (\frac{1}{2}, \frac{1}{2})$

The boundary is $x = \cos\theta$ $y = \sin\theta$ for $0 \le \theta \le 2\pi$

$f|_{bdry} = \cos^2\theta + \sin^2\theta - \cos\theta - \sin\theta + 1$

$= 2 - \cos\theta - \sin\theta$

$\frac{d(f|_{bdry})}{d\theta} = +\sin\theta - \cos\theta$

$\frac{d(f|_{bdry})}{d\theta} = 0$ when $\sin\theta - \cos\theta = 0$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

When $\theta = \frac{\pi}{4}$, the bdry point is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

When $\theta = \frac{5\pi}{4}$, the bdry pt is $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

The bdry of the bdry is $(1,0)$

We must compute

$f(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4} + \frac{1}{4} - \frac{1}{2} - \frac{1}{2} + 1 = \frac{1}{2}$

$f(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = 1 - \sqrt{2} + 1 = 2 - \sqrt{2}$

$f(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = 1 + \sqrt{2} + 1 = 2 + \sqrt{2}$

$f(1,0) = 1 - 1 + 1 = 1$

The maximum of f on our domain is $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 2+\sqrt{2}\right)$
The minimum of f on our domain is $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

② $f_x = x^2 + y + 13 - 6x$

$f_y = \frac{y}{4} + x - 1$

$f_x = f_y = 0$ when $y = 2 - 2x$ and $x^2 + (2-2x) + 13 - 6x = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3, y = -4$ or $x = 5, y = -8$

$f_{xx} = 2x - 6$

$f_{xy} = 1$

$f_{yy} = \frac{1}{2}$

at $(3,-4)$

$\left(f_{xx} f_{yy} - f_{xy}^2\right)\Big|_{(3,-4)} = 0 - 1 < 0 \quad \therefore \boxed{(3,-4, f(3,-4)) \text{ is a saddle pt}}$

at $(5,-8)$

$\left(f_{xx} f_{yy} - f_{xy}^2\right)\Big|_{(5,-8)} = 4 \cdot \frac{1}{2} - 1^2 > 0$

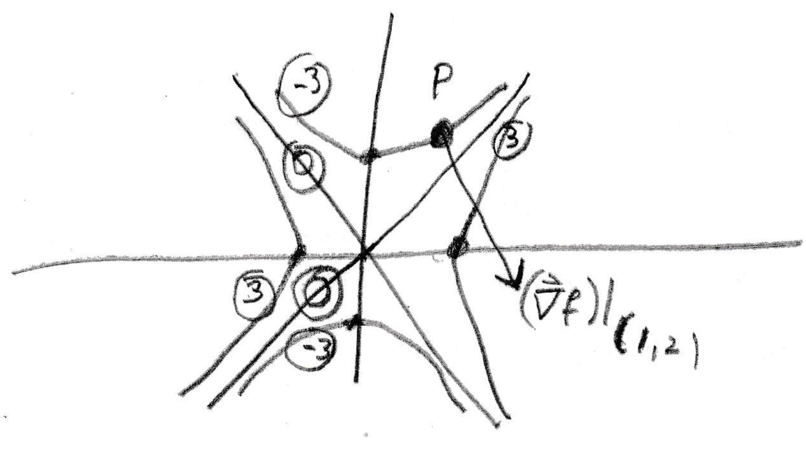
$f_{xx}\Big|_{(5,-8)} = 4 > 0 \quad \therefore \boxed{(5,-8, f(5,-8)) \text{ is a local min.}}$

(3) $3x^2 + 2y^2 - 12x + 4y + 9 = 0$

$3(x^2 - 4x + \boxed{4}) + 2(y^2 + 2y + \triangle\!\!1) = -9 + 3\boxed{4} + 2\triangle\!\!1$

$$\boxed{3(x-2)^2 + 2(y+1)^2 = 5}$$

(4) (a)

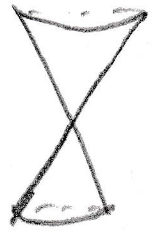


P is $(1,2)$ P is on $-3 = x^2 - y^2$

or $y^2 - x^2 = 3$

(b) $\vec{\nabla} f\big|_{(1,2)} = (2x\vec{i} - 2y\vec{j})\big|_{(1,2)} = 2\vec{i} - 4\vec{j}$

(5) $z^2 = x^2 + y^2$ is a cone



It looks like an hourglass

(6) We want the plane through $(0,1,-2)$ which is perpendicular

to $(2\vec{i}+3\vec{j}+4\vec{k}) \times \overrightarrow{(0,1,-2)(3,4,5)}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 3 & 3 & 7 \end{vmatrix} = (21-12)\vec{i} - (14-12)\vec{j} - 3\vec{k}$$

$$= 9\vec{i} - 2\vec{j} - 3\vec{k}$$

our plane is

$$9(x-0) - 2(y-1) - 3(z+2) = 0$$

or $9x - 2y - 3z + 2 - 6 = 0$

$$\boxed{9x - 2y - 3y = 4}$$

Check The line on the left is on our answer because

$$9(2t+3) - 2(3t+4) - 3(4t+5) = (18-6-12)t + 27 - 8 - 15 = 4 ✓$$

The line on the right ...

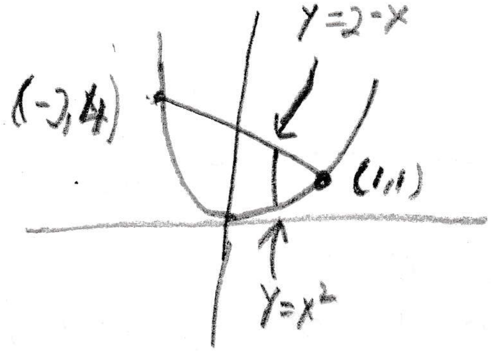
$$9(2t) - 2(3t+1) - 3(4t-2) = (18-6-12)t - 2 + 6 = 4 ✓$$

7 $\text{arclength} = \int_0^4 |\vec{r}\,'(t)|\,dt = \int_0^4 \left|2t\vec{i} + 4t^{\frac{1}{2}}\vec{j} + 4\vec{k}\right| dt$

$= \int_0^4 \sqrt{4t^2 + 16t + 16}\; dt = \int_0^4 \sqrt{(2t+4)^2}\; dt$

$= \int_0^4 2t+4\; dt = t^2 + 4t \Big]_0^4 = 16 + 16 = \boxed{32}$

8



intersection $x + x^2 = 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

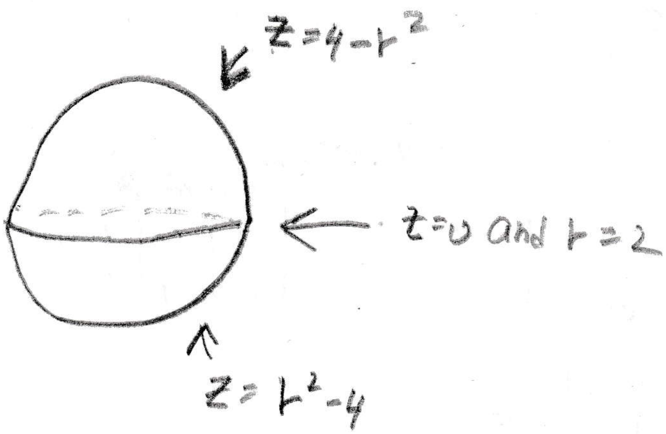
$x = -2,\ x = 1$

$(-2,4)$ $(+1,1)$

$\text{area} = \int_{-2}^{1} \int_{x^2}^{2-x} dy\,dx = \int_{-2}^{1} 2 - x - x^2\; dx = 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \boxed{\frac{9}{2}}$

9



$\text{Vol} = \underbrace{\int\int}_{\text{fattest cross section}} \int_{\text{bot}}^{\text{top}} = \int_0^{2\pi} \int_0^2 \int_{r^2-4}^{4-r^2} r\, dz\, dr\, d\theta$

$= 2\pi \cdot \int_0^2 r(8 - 2r^2)\, dr$

$= 2\pi \int_0^2 (8r - 2r^3)\, dr$

$2\pi \left(4r^2 - \frac{r^4}{2}\right)\Big]_0^2 = \boxed{16\pi}$