

Please PRINT your name \_\_\_\_\_

**No calculators, cell phones, computers, notes, etc.**

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

### Quiz 8, February 17, 2020

**What kind of geometric object is the intersection of the set of all points in 3-space which satisfy  $x + y + 3z = 6$  and the set of all points in 3-space which satisfy  $2x + y + z = 3$ ? Parameterize this object.**

The geometric object is a line. We find two points on the line. When  $x = 0$ , we solve  $y + 3z = 6$  and  $y + z = 3$ . Equation 1 minus equation two gives  $2z = 3$ ; hence  $z = \frac{3}{2}$ . It follows that  $y = 3 - \frac{3}{2} = \frac{3}{2}$ . Observe that  $(0, \frac{3}{2}, \frac{3}{2})$  does satisfy both equations.

When  $z = 0$ , we solve  $x + y = 6$  and  $2x + y = 3$ . Equation 1 minus equation 2 is  $-x = 3$  or  $x = -3$ . We compute that  $y = 6 - x = 6 + 3 = 9$ . Observe that  $(-3, 9, 0)$  does satisfy both equations. The vector from  $(0, \frac{3}{2}, \frac{3}{2})$  to  $(-3, 9, 0)$  is  $\vec{v} = -3\vec{i} + \frac{15}{2}\vec{j} - \frac{3}{2}\vec{k}$ . The line through  $(-3, 9, 0)$  and parallel to  $\vec{v}$  is

$$\begin{cases} x = -3 - 3t \\ y = 9 + \frac{15}{2}t \\ z = 0 - \frac{3}{2}t \end{cases}$$

**Check.** Observe that our proposed answer is always on the first plane:

$$(-3 - 3t) + 9 + \frac{15}{2}t - 3(\frac{3}{2}t) = 6, \checkmark$$

and our proposed line is always on the second plane

$$2(-3 - 3t) + 9 + \frac{15}{2}t - \frac{3}{2}t = 3 \checkmark.$$