Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 7, Monday, March 29, 2021

Find all local minima, local maxima and saddle points for the function $f(x,y) = x^2 + 4y^2 - 6x + 8y - 15$.

Answer: We compute $\frac{\partial f}{\partial x} = 2x - 6$ and $\frac{\partial f}{\partial y} = 8y + 8$. Both partial derivatives are zero at the point $(x,y) = (3,-1)$. We apply the second derivative test at that point. We compute $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y \partial x} = 0$, and $\frac{\partial^2 f}{\partial y^2} = 8$.

Observe that

$$\left(\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial y \partial x} \right)^2 \right) \Bigg|_{(x,y)=(3,-1)} = 16,$$

which is positive. Thus, $(3,-1,f(3,1))$ is not a saddle point; it is either a local maximum or a local minimum. Also

$$\frac{\partial^2 f}{\partial x^2} \Bigg|_{(x,y)=(3,-1)} = 2,$$

which is positive. We conclude that

$$\boxed{(3,-1,f(3,1)) \text{ is a local minimum.}}$$