

Please PRINT your name \_\_\_\_\_

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

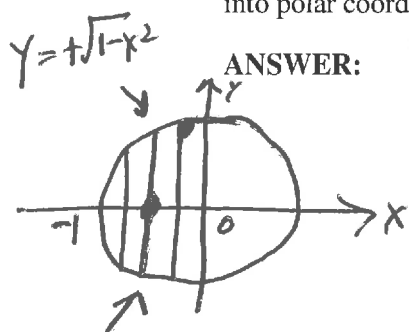
The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 7, November 29, 2018

Convert

$$\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{1+\sqrt{x^2+y^2}} dy dx$$

into polar coordinates and then evaluate the resulting integral.



ANSWER:

For each fixed  $x$  between  $-1$  and  $0$ ,  
 $y$  goes from  $-\sqrt{1-x^2}$  to  $+\sqrt{1-x^2}$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 \frac{2r}{1+r} dr d\theta = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 2 - \frac{2}{1+r} dr d\theta$$

long division gives  $1+r \frac{2}{2r+2}$        $\frac{2r}{r+1} = 2 - \frac{2}{1+r}$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2r - 2 \ln(1+r)]_0^1 d\theta = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (2 - 2 \ln 2) d\theta$$

$$= (2 - 2 \ln 2) \theta \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \boxed{(2 - 2 \ln 2) \pi}$$