Please PRINT your name ____________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, March 28, 2022

Find the absolute extreme points of the function $f(x,y) = x + y - xy$**, which is defined on the closed triangle with vertices at** $(0,0)$**,** $(0,2)$**, and** $(4,0)$**.**

We put a picture of the domain on the last page. We see that the boundary has three pieces. Eventually, we will look at f restricted to each of these three pieces. Eventually, also, we will look at f evaluated at each of the end points of the boundary.

First we look for interior points where both partial derivatives vanish. We compute $f_x = 1-y$ and $f_y = 1-x$. If $f_x = 0$ and $f_y = 0$ then $x = 1$ and $y = 1$. We will study $(1,1)$ in our final step.

Now we look at f restricted to the vertical line $x = 0$, with $0 \le y \le 2$. This function is

$$f|_{x=0} = y.$$

We see that $\frac{d}{dy}(f|_{x=0}) = 1$, which is never zero. Thus, the extreme points of $f|_{x=0}$ occur at the end points $(0,0)$ and $(0,2)$. We already know to study these points in our final step.

Now we look at f restricted to the horizontal line $y = 0$, with $0 \le x \le 4$. This function is

$$f|_{y=0} = x.$$

We see that $\frac{d}{dx}(f|_{y=0}) = 1$, which is never zero. Thus, the extreme points of $f|_{y=0}$ occur at the end points $(0,0)$ and $(4,0)$. We already know to study these points in our final step.

Now we look at f restricted to the slanting line $y = -\frac{1}{2}x + 2$, with $0 \le x \le 4$. This function is

$$f|_{y=-\frac{1}{2}x+2} = x + (-\tfrac{1}{2}x + 2) - x(-\tfrac{1}{2}x + 2) = \tfrac{x^2}{2} - \tfrac{3}{2}x + 2.$$

We compute

$$\tfrac{d}{dx}(f|_{\text{slanting line}}) = x - \tfrac{3}{2}.$$

Thus, $\frac{d}{dx}(f|_{\text{slanting line}}) = 0$ when $x = \frac{3}{2}$ and $y = 2 - \frac{3}{4} = \frac{5}{4}$.

It is time for the final step. The extreme points of f on our domain occur at one of the points $(0,0)$, $(0,2)$, $(4,0)$, $(1,1)$, or $(\frac{3}{2}, \frac{5}{4})$. We evaluate f at these 5 points; the largest answer is the maximum. The smallest answer is the minimum.

$$f(0,0) = 0$$
$$f(0,2) = 2$$
$$f(4,0) = 4$$
$$f(1,1) = 1$$
$$f(\tfrac{3}{2}, \tfrac{5}{4}) = \tfrac{7}{8}$$

We conclude that $(4,0,4)$ is the maximum of f on our domain and $(0,0,0)$ is the minimum of f on our domain.

The triangle with vertices (0,0), (0,2) and (4,0)