No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

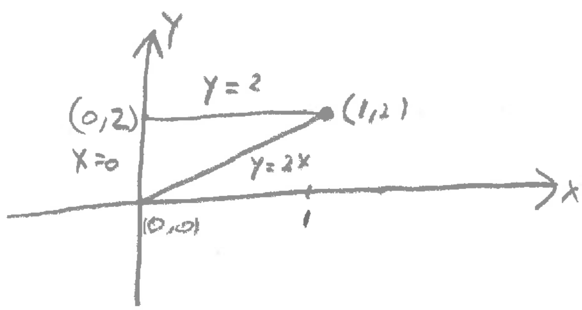
Quiz 6, Nov. 2, 2017, 11:40 class

Find the absolute maxima and absolute minima of the function

$$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$$

on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, and $y = 2x$ in the first quadrant.

ANSWER:



First we look for all points P in the interior with $f_x(P) = 0$ and $f_y(P) = 0$. We compute $f_x = 4x - 4 + 2y - 4$ and $f_y = 2y - 4$. If $f_x = 0$ and $f_y = 0$, then

$$\begin{cases} 4x - 4 + 2y - 4 = 0 \\ 2y - 4 = 0 \end{cases}$$

$$\begin{cases} 4x = 8 - 2y \\ y = 2 \end{cases}$$

$$\begin{cases} x = 1 \\ y = 2 \end{cases}$$

We consider $(1,2)$.

Now we look at points where the derivative of (f restriced to the boundary) is zero.

The first part of the the boundary is $x = 0$ with $0 \le y \le 2$. On this part of the boundary, $f(y) = y^2 - 4y + 1$. We compute $f'(y) = 2y - 4$. We see that $f'(y) = 0$ when $y = 2$. So, we consider the point $(0,2)$.

The second part of the boundary is $y = 2$ with $0 \le x \le 1$. On this part of the boundary, $f(x) = 2x^2 - 4x + 4 - 8 + 1$, so $f'(x) = 4x - 4$. We see that $f'(x) = 0$ when $x = 1$. We consider the point $(1,2)$.

The third part of the boundary is $y = 2x$ with $0 \leq x \leq 1$. On this part of the boundary, $f(x) = 2x^2 - 4x + (2x)^2 - 4(2x) + 1$; so $f(x) = 6x^2 - 12x + 1$. We compute $f'(x) = 12x - 12$. We see that $f'(x) = 0$ when $x = 1$. We consider the point $(1,2)$ (again).

Finally, we must consider the endpoints on the boundary. They are $(0,0)$, $(0,2)$, and $(1,2)$.

The maximum and the minimum of f on the given region are contained in the following list:

$f(0,0) = 1$	maximum
$f(0,2) = 4 - 8 + 1 = -3$	
$f(1,2) = 2 - 4 + 4 - 8 + 1 = -5$	minimum

The maximum of f on the given region is 1 and $f(0,0) = 1$.
The minimum of f on the given region is -5 and $f(1,2) = -5$.