Please PRINT your name ______________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete, and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 5, March 26, 2019

Find the points on the curve $xy^2 = 54$ which are closest to the origin.

ANSWER: The distance from the point (x,y) to the origin is $\sqrt{x^2+y^2}$. Observe that $\sqrt{x^2+y^2}$ is minimized when x^2+y^2 is minimized.

We want to minimize the function $f(x,y) = x^2+y^2$, subject to the constraint $g = 54$, where $g(x,y) = xy^2$. Thus, we find all points on $g = 54$ with $\vec{\nabla} f = \lambda \vec{\nabla} g$ for some number λ. The equation $\vec{\nabla} f = \lambda \vec{\nabla} g$ is the same as

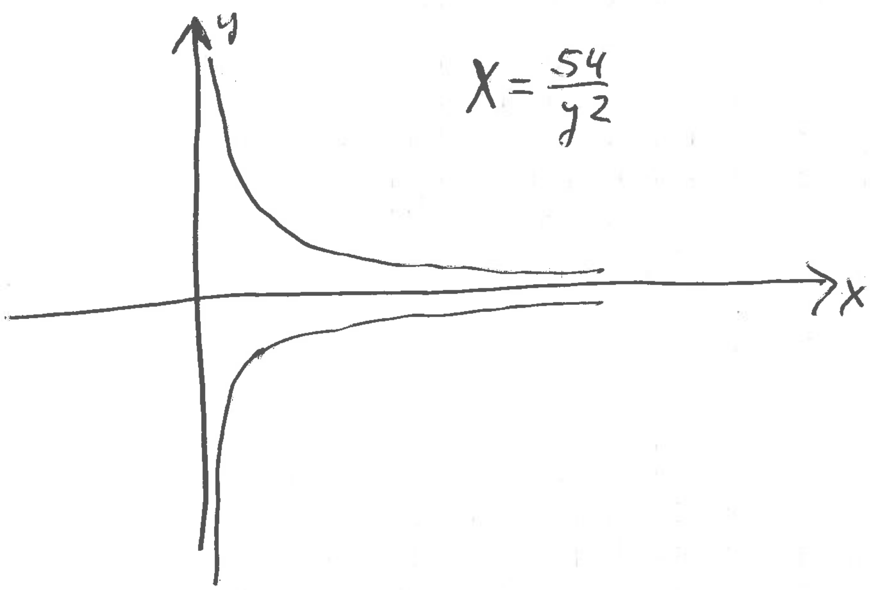
$$2x\vec{i} + 2y\vec{j} = \lambda(y^2\vec{i} + 2xy\vec{j}).$$

We solve

$$\begin{cases} 2x = \lambda y^2 \\ 2y = 2xy\lambda \\ 54 = xy^2 \end{cases}$$

simultaneously. Observe that neither x nor y is zero; hence $1/x = \lambda$ (from the second equation); $2x = (1/x)y^2$ or $2x^2 = y^2$ (from the first equation) and $54 = 2x^3$ from the third equation. Thus, $3 = x$ and $y = \pm 3\sqrt{2}$. A quick glance at the graph of $54 = xy^2$ indicates that there is no point on this curve which is furthest from the origin and that

$\boxed{(3, 3\sqrt{2}) \text{ and } (3, -3\sqrt{2}) \text{ are the points on the curve } 54 = xy^2 \text{ which are closest to the origin.}}$



$X = \dfrac{54}{y^2}$

$\lim_{y \to 0^+} X = +\infty$

$\lim_{y \to 0^-} X = +\infty$

$\lim_{y \to \infty} X = 0$

$\lim_{y \to -\infty} X = 0$