Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. $\boxed{\text{Make your work correct, complete and coherent.}}$

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will return your quiz when I next see you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

<div align="center">

Math 241, Quiz 3, January 29, 2025

</div>

Express $\vec{v} = \vec{i} + 7\vec{j}$ as the sum of a vector parallel to $\vec{b} = \vec{i} + 2\vec{j}$ and a vector perpendicular to $\vec{b}$. **Please make sure that your answer is correct.**

Answer: There is a picture on the next page. We compute

$$\text{proj}_{\vec{b}}\,\vec{v} = \frac{\vec{v} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\,\vec{b}$$
$$= \frac{(\vec{i} + 7\vec{j}) \cdot (\vec{i} + 2\vec{j})}{(\vec{i} + 2\vec{j}) \cdot (\vec{i} + 2\vec{j})}(\vec{i} + 2\vec{j})$$
$$= \frac{15}{5}(\vec{i} + 2\vec{j})$$
$$= 3\vec{i} + 6\vec{j}.$$

We also compute

$$\vec{v} - \text{proj}_{\vec{b}}\,\vec{v} = (\vec{i} + 7\vec{j}) - (3\vec{i} + 6\vec{j}) = -2\vec{i} + \vec{j}.$$

We conclude that

$$\boxed{\begin{array}{l} \vec{v} = (3\vec{i} + 6\vec{j}) + (-2\vec{i} + \vec{j}), \text{ with } 3\vec{i} + 6\vec{j} \text{ parallel to } \vec{b} \\ \text{and } -2\vec{i} + \vec{j} \text{ perpendicular to } \vec{b} \end{array}}$$

Check. Observe that
- $(3\vec{i} + 6\vec{j}) + (-2\vec{i} + \vec{j}) = 1\vec{i} + 7\vec{j} = \vec{v}\checkmark$,
- $3\vec{i} + 6\vec{j}$, which is equal to 3 times $\vec{b}$ is parallel to $\vec{b}$ $\checkmark$, and
- $(-2\vec{i} + \vec{j}) \cdot \vec{b} = (-2\vec{i} + \vec{j}) \cdot (\vec{i} + 2\vec{j}) = 0.\checkmark$

$\vec{v}$

$\vec{v} - \text{Proj}_{\vec{b}} \, \vec{v}$

$\vec{b}$

$\text{Proj}_{\vec{b}} \, \vec{v}$