Please PRINT your name ____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work **correct**, **complete** and **coherent**.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 19, November 13, 2019

Find the absolute maxima and absolute minima of the function

$$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$$

on the closed triangle bounded by the lines $x = 0$, $y = 2$, and $y = 2x$ in the first quadrant.

ANSWER: A picture of the domain of f may be found a different page. The extreme points of f occur either at an interior point where both partial derivatives vanish or at a point on the boundary. There are three pieces of the boundary; we must consider f restricted to each of these pieces. The boundary has three end points. We must consider each of these.

We compute $f_x = 4x - 4$ and $f_y = 2y - 4$. Observe that both partial derivatives vanish at the point $(1,2)$. This point is on the boundary of our region.

The end points of the boundary are $(0,0)$, $(0,2)$, $(1,2)$.

When f is restricted to $x = 0$, the resulting function is $f(y) = y^2 - 4y + 1$. Observe that $f'(y) = 2y - 4$. This function is zero when $y = 2$. We need to consider $(0,2)$ (which is already on our list).

When f is restricted to $y = 2$, the resulting function is $f(x) = 2x^2 - 4x + 2^2 - 4(2) + 1$. The derivative is $f'(x) = 4x - 4$. The derivative is zero, when $x = 1$. The resulting point is $(1,2)$, which is already on our list.

When f is restricted to $y = 2x$, the resulting function is $f(x) = 2x^2 - 4x + (2x)^2 - 4(2x) + 1$. The derivative is $f'(x) = 4x - 4 + 8x - 8$. In other words, $f'(x) = 12x - 12$. Observe that $f'(x) = 0$ when $x = 1$. The resulting point is $(1,2)$, which is already on our list.

The absolute extreme points of $f(x,y)$ on our domain occur at

$$(0,0), \quad (0,2), \quad \text{or} \quad (1,2).$$

Plug these points into f:

$$f(0,0) = 1$$
$$f(0,2) = 4 - 8 + 1 = -3$$
$$f(1,2) = 2 - 4 + 4 - 8 + 1 = -5$$

The absolute maximum of f on our domain occurs at (0,0,1).
The absolute minimum of f on our domain occurs at (1,2,-5).