Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work **correct**, **complete** and **coherent**.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 18, November 11, 2019

Find all the local maxima, local minima, and saddle points of $f(x,y) = x^3 - y^3 - 2xy + 6$.
ANSWER: We first compute

$$f_x = 3x^2 - 2y \quad \text{and} \quad f_y = -3y^2 - 2x.$$

We look for all points P with $f_x(P) = 0$ and $f_y(P) = 0$. So we solve

$$\begin{cases} 3x^2 - 2y = 0 \\ -3y^2 - 2x = 0 \end{cases}$$

simultaneously:

$$\begin{cases} (3/2)x^2 = y \\ -3y^2 - 2x = 0 \end{cases}$$

$$\begin{cases} (3/2)x^2 = y \\ -3((3/2)x^2)^2 - 2x = 0 \end{cases}$$

$$\begin{cases} (3/2)x^2 = y \\ -(27/4)x^4 - 2x = 0 \end{cases}$$

$$\begin{cases} (3/2)x^2 = y \\ -x(27x^3 + 8) = 0 \end{cases}$$

So either $x = -2/3$ and $y = (3/2)(2/3)^2 = 2/3$ or $x = 0$ and $y = 0$. The two points where both partial derivatives vanish are $P_1 = (-2/3, 2/3)$ and $P_2 = (0,0)$. We apply the second derivative test at these points. We compute

$$f_{xx} = 6x, \quad f_{xy} = -2 \quad f_{yy} = -6y.$$

We see that

$$f_{xx}(P_1)f_{yy}(P_1) - [f_{xy}(P_1)]^2 = 6(-2/3)(-6)(2/3) - (-2)^2 = 16 - 4 = 12$$

and $f_{xx}(P_1) = 6(-2/3) = -4$. Thus $0 < f_{xx}(P_1)f_{yy}(P_1) - [f_{xy}(P_1)]^2$ and $f_{xx}(P_1) < 0$. We conclude that

$$\boxed{(-2/3, 2/3, f(-2/3, 2/3)) \text{ is a local maximum of } f}.$$

We also see that

$$f_{xx}(P_2)f_{yy}(P_2) - [f_{xy}(P_2)]^2 = 0 - (-2)^2 = -4 < 0.$$

We conclude that

$$\boxed{(0,0,0) \text{ is a saddle point of } f}.$$