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## No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.
The quiz is worth 5 points. The solutions will be posted on my website later today.

## Quiz 17, November 4, 2019

An object starts at the origin with velocity $4 \vec{i}+8 \vec{j}$. The acceleration of the object at time $t$ is $\overrightarrow{\boldsymbol{r}}^{\prime \prime}(t)=2 e^{t} \overrightarrow{\boldsymbol{i}}+16 e^{2 t} \overrightarrow{\boldsymbol{j}}$. What is the $x$-coordinate of the object when the $y$-coordinate is 12 ?
ANSWER:
Integrate to learn that $\overrightarrow{\boldsymbol{r}}^{\prime}(t)=2 e^{t} \overrightarrow{\boldsymbol{i}}+8 e^{2 t} \overrightarrow{\boldsymbol{j}}+\overrightarrow{\boldsymbol{c}}_{1}$. Plug in $t=0$ to learn

$$
4 \overrightarrow{\boldsymbol{i}}+8 \overrightarrow{\boldsymbol{j}}=\overrightarrow{\boldsymbol{r}}^{\prime}(0)=2 \overrightarrow{\boldsymbol{i}}+8 \overrightarrow{\boldsymbol{j}}+\overrightarrow{\boldsymbol{c}}_{1}
$$

So, $\overrightarrow{\boldsymbol{c}}_{1}=2 \overrightarrow{\boldsymbol{i}}$,

$$
\overrightarrow{\boldsymbol{r}}^{\prime}(t)=\left(2 e^{t}+2\right) \overrightarrow{\boldsymbol{i}}+8 e^{2 t} \overrightarrow{\boldsymbol{j}}
$$

Integrate again to learn

$$
\overrightarrow{\boldsymbol{r}}(t)=\left(2 e^{t}+2 t\right) \overrightarrow{\boldsymbol{i}}+4 e^{2 t} \overrightarrow{\boldsymbol{j}}+\overrightarrow{\boldsymbol{c}}_{2} .
$$

Plug in $t=0$ to learn

$$
0=\overrightarrow{\boldsymbol{r}}(0)=2 \overrightarrow{\boldsymbol{i}}+4 \vec{j}+\overrightarrow{\boldsymbol{c}}_{2}
$$

So, $\overrightarrow{\boldsymbol{c}}_{2}=-2 \overrightarrow{\boldsymbol{i}}-4 \overrightarrow{\boldsymbol{j}}$ and

$$
\overrightarrow{\boldsymbol{r}}(t)=\left(2 e^{t}+2 t-2\right) \overrightarrow{\boldsymbol{i}}+\left(4 e^{2 t}-4\right) \overrightarrow{\boldsymbol{j}}
$$

The $y$-coordinate of the object is 12 when $4 e^{2 t}-4=12$; thus, $4 e^{2 t}=16$;

$$
\begin{gathered}
e^{2 t}=4 \\
2 t=\ln 4 \\
t=(\ln 4) / 2=(2 \ln 2) / 2=\ln 2
\end{gathered}
$$

The $x$-coordinate of the object is $2 e^{\ln 2}+2 \ln 2-2=4+\ln 4-2=2+\ln 4$ when the $y$ coordinate of the object is 12 .

