

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 13, Wednesday, November 9, 2020

Find all local maxima, local minima, and saddle points of

$$f(x,y) = 2x^2 + 3xy + 4y^2 - 5x + 2y.$$

Answer: We compute

$$f_x = 4x + 3y - 5 \quad \text{and} \quad f_y = 3x + 8y + 2.$$

We solve

$$\begin{cases} 4x + 3y - 5 = 0 \\ 3x + 8y + 2 = 0 \end{cases}$$

simultaneously. The solution set is unchanged if we replace Equation 2 with Equation 2 minus $3/4$ Equation 1. We solve

$$\begin{cases} 4x + 3y - 5 = 0 \\ \frac{23}{4}y + \frac{23}{4} = 0 \end{cases}$$

simultaneously.

The solution set is unchanged if we replace Equation 2 with $\frac{4}{23}$ Equation 2. We solve

$$\begin{cases} 4x + 3y - 5 = 0 \\ y + 1 = 0 \end{cases}$$

simultaneously. So, $y = -1$ and $x = 2$. We apply the second derivative test at this point.

Calculate $f_{xx} = 4$, $f_{xy} = 3$, and $f_{yy} = 8$. Thus, $H = f_{xx}f_{yy} - f_{xy}^2 = 4(8) - 3^2$, which is positive. Observe $f_{xx} = 4$, which is also positive. We conclude that

$$\boxed{(2, -1, f(2, -1)) \text{ is a local minimum.}}$$