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12. Identify all local extreme points and all saddle points of
$f(x,y) = x^2y - 6y^2 - 3x^2$.

$f_x = 2xy - 6x$

$f_y = x^2 - 12y$

both partials vanish

at $y = \frac{x^2}{12}$

$0 = 2x\,\frac{x^2}{12} - 6x$

$0 = \frac{x}{6}(x^2 - 36)$

$x = 0,\ 6,\ -6$

$y = 0,\ 3,\ 3$

our critical pts are

$(0,0)\ (6,3)\ (-6,3)$

$f_{xx} = 2y - 6$

$f_{xy} = 2x$

$f_{yy} = -12$

at $(0,0)$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 6 \cdot 12 > 0$

$f_{yy} < 0$

$\therefore (0,0)$ is a loc. max

at $(6,3)$

$D = 0 \cdot (-12) - (12)^2 < 0$

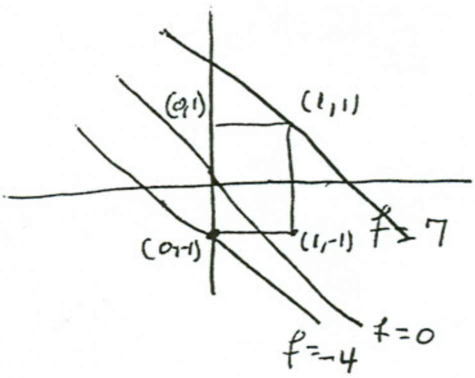
$\therefore (6,3)$ is a saddle pt.

at $(-6,3)$

$D = 0(-12) - (12)^2 < 0$

$\therefore (-6,3)$ is a saddle pt.

13. Find the global extreme points of $f(x,y) = 3x + 4y$, which is defined on
$S = \{(x,y) \mid 0 \le x \le 1, -1 \le y \le 1\}$.



$f_x = 3$

$f_y = 4$

f_x and f_y are never 0

when $x=0$ $f = 4y$
$f' = 4$ never 0

when $y = 1$ $f = 3x+4$
$f' = 3$ never 0

when $x = 1$ $f = 3+4y$
$f' = 4$ never 0

when $y = -1$ $f = 3x-4$ f' never 0

we only need consider the corners

$f(0,-1) = -4$

$f(1,-1) = -1$

$f(1,1) = 7$

$f(0,1) = 4$

Max is $(1,1,7)$

min is $(0,-1,-4)$