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4. Find the area inside $r = 4\sin\theta$.

This is the circle $r^2 = 4r\sin\theta$

$x^2+y^2 = 4y$

$x^2+(y-2)^2 = 4$

Its area is $\pi r^2 = \pi 2^2 = 4\pi$

5. Find the volume of the solid which is bounded by $z = x^2 + y^2$ and $z = \sqrt{1-x^2-y^2}$.

intersection is $z = r^2$, $z = \sqrt{1-r^2}$

$r^2 = \sqrt{1-r^2}$

$r^4 = 1-r^2$

$r^4+r^2-1=0$

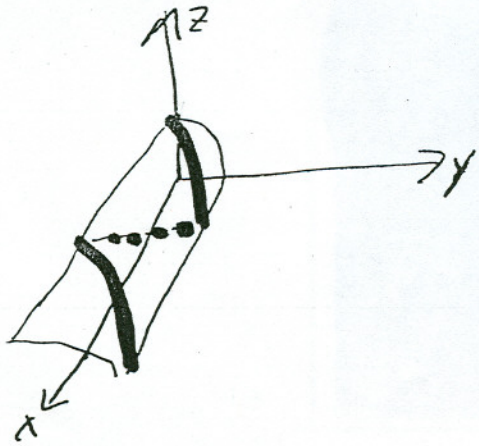
$r^2 = \frac{-1\pm\sqrt{1+4}}{2}$ $\quad r = \sqrt{\frac{\sqrt{5}-1}{2}}$

$$\int_0^{2\pi}\int_0^{\sqrt{\frac{\sqrt{5}-1}{2}}}\int_{r^2}^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta = 2\pi\int_0^{\sqrt{\frac{\sqrt{5}-1}{2}}} -r^3+r\sqrt{1-r^2}\,dr$$

$$= 2\pi\left(-\frac{r^4}{4}+\frac{2}{3}\left(\frac{1}{2}\right)(1-r^2)^{\frac{3}{2}}\right)\Bigg]_0^{\sqrt{\frac{\sqrt{5}-1}{2}}}$$

$$= 2\pi\left(-\frac{\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^4}{4} - \frac{1}{3}\left(1-\frac{\sqrt{5}-1}{2}\right)^{\frac{3}{2}}+\frac{1}{3}\right)$$

6. Describe and graph $\vec{r}(t) = t\,\vec{i} + \sin t\,\vec{j} + \cos t\,\vec{k}$.



A helix on a cylinder whose center is the x-axis