

# MATH 241 Spring 1998, final

~~STUDY~~  
~~TEST~~  
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PRINT Your Name: \_\_\_\_\_

There are 15 problems on 6 pages. Each problem is worth 10 points. SHOW your work. **CIRCLE** your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Find the equations of the line which contains the points  $(4, 2, -3)$  and  $(4, -3, 0)$ . The vector which connects the points is  $5\vec{i} - 3\vec{j}$

line  $\begin{cases} x = 4 \\ y = 5t - 3 \\ z = -3t \end{cases}$

2. Find the equation of the plane which contains the points  $(4, 2, -3)$ ,  $(4, -3, 0)$ , and  $(2, 3, 4)$ .

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -5 & 3 \\ -2 & 1 & 7 \end{vmatrix}$$

$$= -38\vec{i} - 6\vec{j} - 10\vec{k}$$

$$= -38(x-4) - 6(y+3) - 10z$$

$$0 = -19(x-4) + 3(y+3) + 5z$$

$19x + 3y + 5z = 67$

3. Let  $\vec{a} = 4\vec{i} + 2\vec{k}$  and  $\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$ . Find vectors  $\vec{u}$  and  $\vec{v}$  with  $\vec{b} = \vec{u} + \vec{v}$ ,  $\vec{u}$  parallel to  $\vec{a}$ , and  $\vec{v}$  perpendicular to  $\vec{a}$ .



$$\vec{u} = \text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a} = \frac{8+6}{16+4} \vec{a} = \frac{14}{20} (4\vec{i} + 2\vec{k}) = \frac{7}{10} (4\vec{i} + 2\vec{k}) = \frac{14}{10} \vec{i} + \frac{14}{10} \vec{k}$$

$$\vec{v} = \vec{b} - \vec{u} = \frac{1}{10} (20\vec{i} - 10\vec{j} + 30\vec{k} - 28\vec{i} - 14\vec{k})$$

$\vec{v} = \frac{1}{10} (-8\vec{i} - 10\vec{j} + 16\vec{k})$